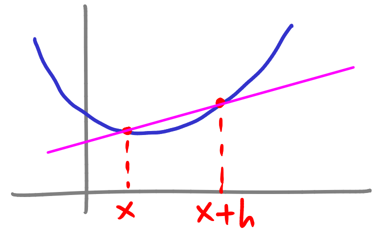
Lecture 9

19 Sep 2018

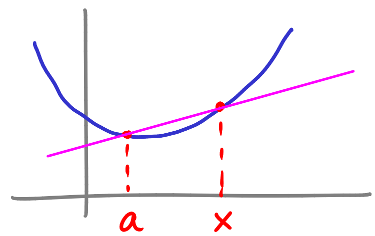
My office hour: M 2-3
W 3:30–4:30 RLM 9.134
today

Last time: <u>derivatives</u>

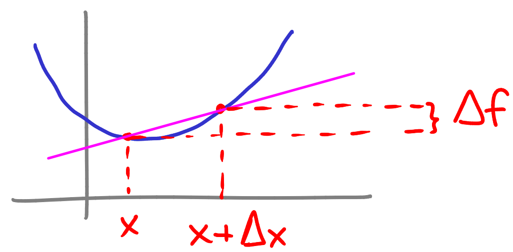
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



or $$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$



or $$f'(x) = \lim_{\Delta x\to 0} \frac{\Delta f}{\Delta x}$$

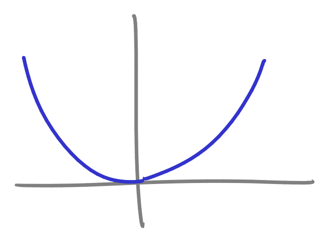


We say $f(x)$ is differentiable at a if $f'(a)$ exists

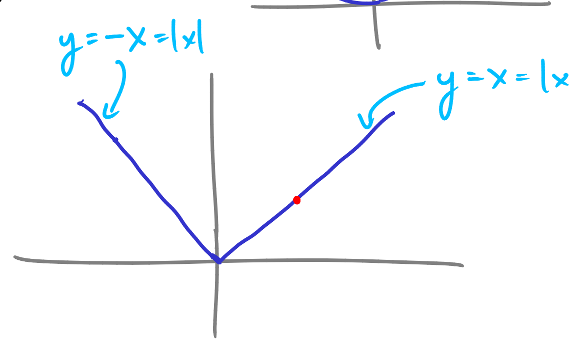
(ie if $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ exists)

<u>Ex</u> $f(x)=x^2$ is differentiable at all real #'s a,

because $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ <u>exists</u> (and equals $2a$)



<u>Ex</u> Where is $f(x)=|x|$ differentiable?

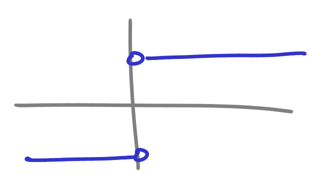


If $x>0$, $f'(x)=1$ (exists)

If $x<0$, $f'(x)=-1$ (exists)

If $x=0$, let's look closer:

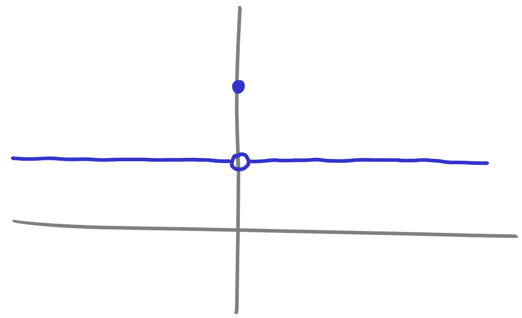
$$f'(0) = \lim_{h\to 0} \frac{f(h)-f(0)}{h} = \lim_{h\to 0} \frac{|h|-|0|}{h} = \lim_{h\to 0} \frac{|h|}{h} \quad \underline{\underline{\text{DNE}}}$$



So f is not differentiable at $x=0$. (and is diff'ble at all other x.)

(In general, graph of f has sharp corner at $x \Rightarrow f$ is not diff'ble at x.)

<u>Ex</u> Where is $f(x) = \begin{cases} 1 & x \neq 0 \\ 2 & x = 0 \end{cases}$ differentiable?



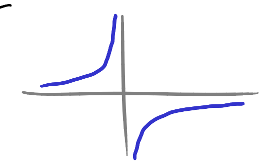
At any $x \neq 0$, $f'(x) = 0$ (exists)

At $x=0$,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(0) = \lim_{h\to 0} \frac{f(h)-f(0)}{h}$$

plug in small h: $(h\neq 0)$ $\quad \dfrac{f(h)-f(0)}{h} = \dfrac{1-2}{h} = \dfrac{-1}{h}$ DNE

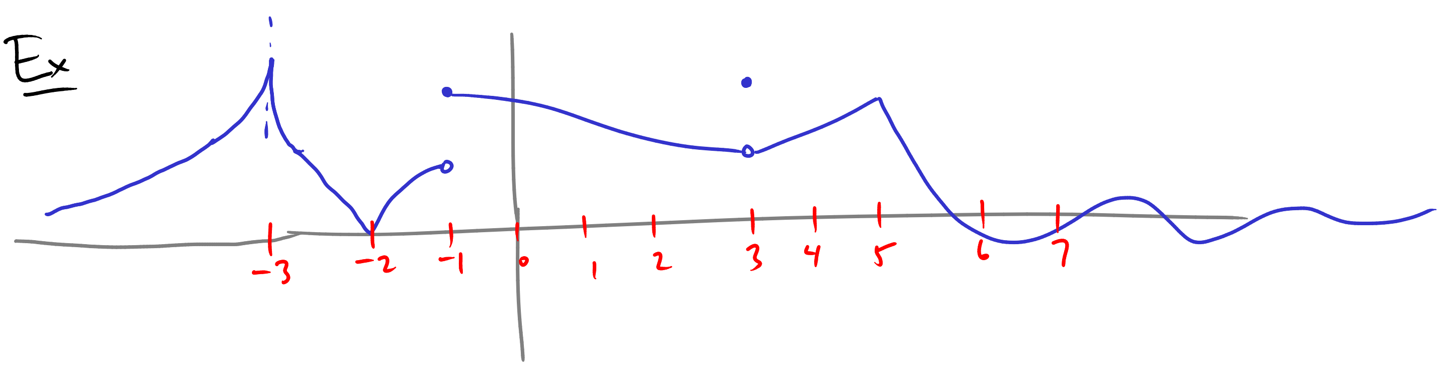


$\left[\lim_{h\to 0} -\frac{1}{h} \text{ DNE} \right]$

So $f(x)$ is not diff'ble at $x=0$. ($\longrightarrow f(x)$ is diff'ble at all x except 0.)

In general, where f is not continuous, f is not differentiable.

<u>Ex</u>



f is differentiable except at $-3, -2, -1, 3, 5$

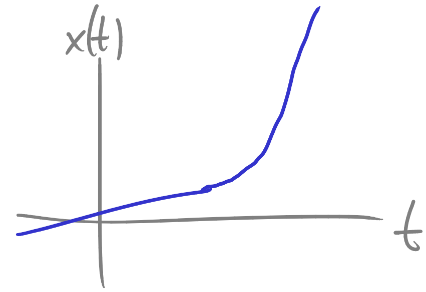
(<u>Rk</u> At points where $\displaystyle\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \pm\infty$ we say f is not differentiable)

Interpretation of $f'(x)$

① If $x(t)$ is the position of an object at time t

then $x'(t)$ is the velocity of the object.

$$x'(t) = v(t)$$



Ex An electron in a uniform electric field moves with

$$x(t) = \tfrac{1}{2}t^2$$

What is its velocity at time t?

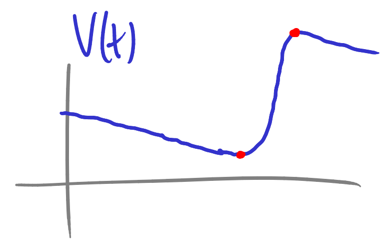
$$v(t) = x'(t) = \lim_{h\to 0} \frac{x(t+h) - x(t)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2}(t+h)^2 - \frac{1}{2}t^2}{h} = \cdots = \underline{t}$$

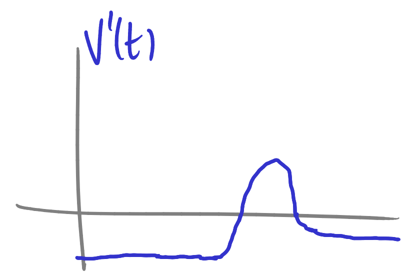
② In general if t = time

$f'(t)$ is the rate of change of $f(t)$.

Ex If $V(t)$ = volume of water in Lake Travis at time t (in gal) (in sec)



$V'(t)$ = rate of change of the volume (in gal/sec)



We can also repeat:

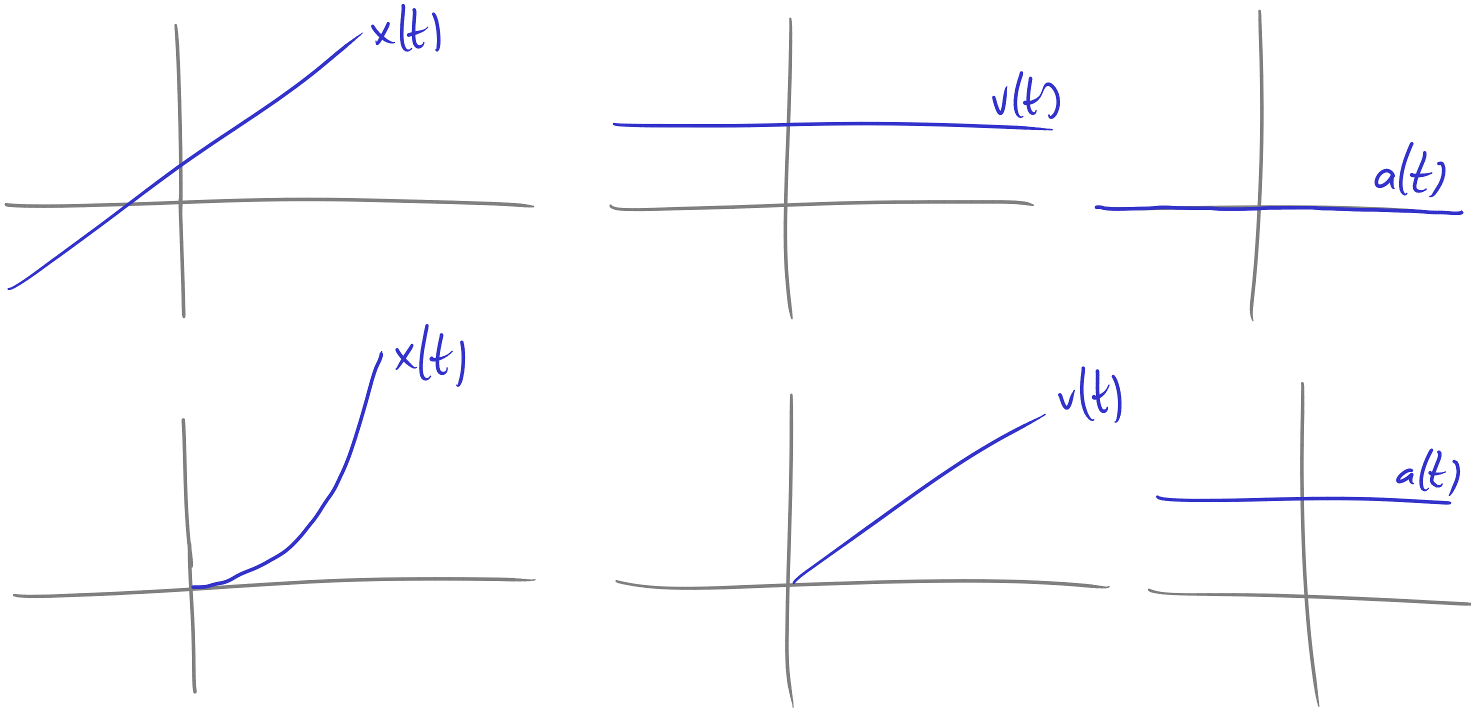
$$f''(x) = \text{derivative of } f'(x) = \lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h}$$

"second derivative"

If $x(t)$ is position

$x'(t) = v(t)$ is velocity

$x''(t) = v'(t) = a(t)$ is acceleration (rate of change of the velocity)



Another notation:

$$\frac{df}{dx} \quad \text{or} \quad \frac{d}{dx}f(x) \quad \text{means} \quad f'(x) \qquad \left(\approx \frac{\Delta f}{\Delta x}\right)$$

$$\frac{d^2f}{dx^2} \quad \text{or} \quad \frac{d^2}{dx^2}f(x) \quad \text{means} \quad f''(x)$$

$$\vdots$$

$$\frac{d^nf}{dx^n} \quad \text{or} \quad \frac{d^n}{dx^n}f(x) \quad \text{means} \quad f^{\overset{n\text{ times}}{''''\cdots}}(x) = f^{(n)}(x)$$

Ex $\frac{d}{dx}(x^2) = 2x \qquad \frac{d^2}{dx^2}(x^2) = \frac{d}{dx}(2x) = 2$