Lecture 13 — 28 Sep 2018

I have extra office hour today, 4–5 (RLM 9.134)

Midterm 1 Monday Oct 1 in class (come on time w/ pencil, eraser, ID, ~~calculator~~)

Last time: chain rule

Ex
$$y(t) = \left(\frac{t-2}{2t+1}\right)^9 = 9\left(\frac{t-2}{2t+1}\right)^8 \frac{d}{dt}\left(\frac{t-2}{2t+1}\right)$$
$$= 9\left(\frac{t-2}{2t+1}\right)^8 \frac{(2t+1)(1)-(t-2)(2)}{(2t+1)^2}$$
$$= 9\left(\frac{t-2}{2t+1}\right)^8 \frac{2t+1-2t+4}{(2t+1)^2}$$
$$= 9\left(\frac{t-2}{2t+1}\right)^8 \frac{5}{(2t+1)^2} = 45\frac{(t-2)^8}{(2t+1)^{10}}$$

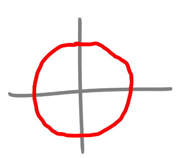
Implicit Differentiation

So far we looked at graphs of functions $y(x)$ usually given by some definite formula,

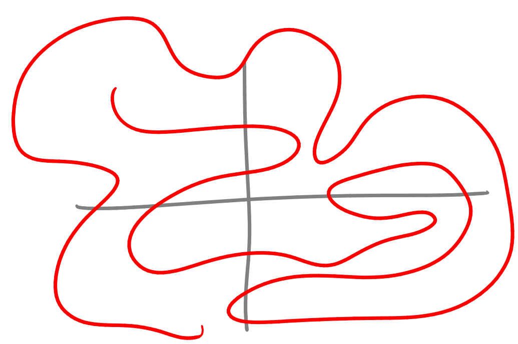
like $y = e^{x\cos\left(\frac{x-1}{4}\right)}$

Sometimes we know a relation between y and x but not a formula for y.

Ex $y^2+x^2=4$



Ex $y^7 + 8yx^3 + 17y^2x^8 = 0$ $(*)$



Even without a formula for y we can still find the slope of the tangent line at some given (x,y)!

How? Locally there is a function $y(x)$ giving this graph

Then apply $\frac{d}{dx}$ to our eq. $(*)$:

$$\frac{d}{dx}\left(y^7 + 8yx^3 + 17y^2x^8\right) = \frac{d}{dx}(0) = 0$$

$$7y^6\frac{dy}{dx} + 8x^3\frac{dy}{dx} + 8y\cdot 3x^2 + 17\cdot 2y\frac{dy}{dx}\cdot x^8 + 17y^2\cdot 8x^7 = 0$$

$$\frac{dy}{dx}\left(7y^6 + 8x^3 + 34y\cdot x^8\right) + 24y\cdot x^2 + 136y^2x^7 = 0$$

$$y'\left(7y^6 + 8x^3 + 34yx^8\right) = -24yx^2 - 136y^2x^7$$

$$\underline{\underline{y' = \frac{-24yx^2 - 136y^2x^7}{7y^6 + 8x^3 + 34yx^8}}}$$

plug in (x,y) on the graph to get the slope

Ex $\quad x^2 + y^2 = 4$

$$\frac{d}{dx}(x^2 + y^2) = 0$$

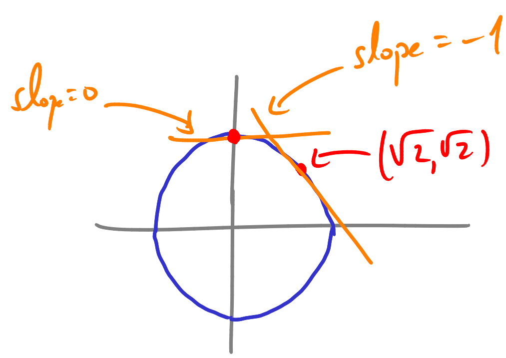
$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

e.g. at $x = \sqrt{2}$, $y = \sqrt{2}$ $\qquad \frac{dy}{dx} = \frac{-\sqrt{2}}{\sqrt{2}} = -1$

at $x = 0$, $y = 1$ $\qquad \frac{dy}{dx} = \frac{0}{1} = 0$



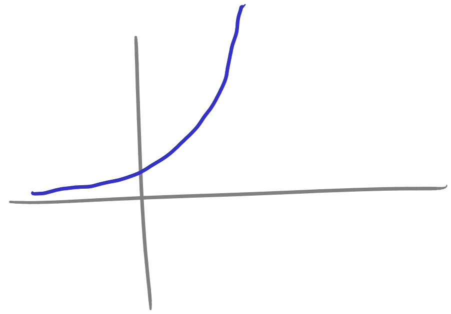
Recall $\quad \frac{d}{dx}(x^r) = rx^{r-1}$ except if $r = 0$ $\qquad \frac{d}{dx}x^3 = 3x^2 \quad \frac{d}{dx}x^{1/2} = \frac{1}{2}x^{-1/2}$

How to get a function whose derivative is x^{-1}?

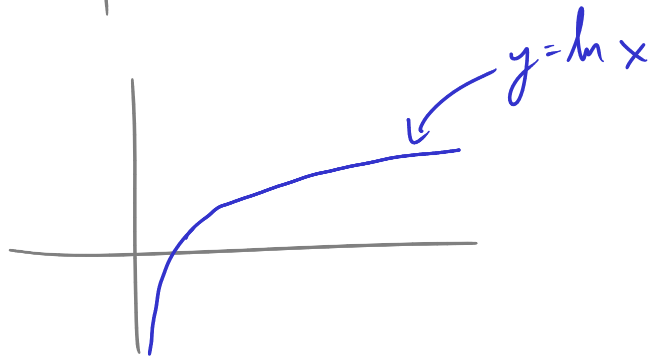
(a funny exception...)

Logarithms

Recall $y = e^x$



and its inverse, $y = \ln x$



What is the derivative of $y = \ln x$?

Write $\quad e^y = x$

$$\frac{d}{dx}(e^y) = \frac{d}{dx}(x)$$

$$e^y \frac{dy}{dx} = 1$$

$$x \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{x}$$

So: $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$.

Ex $\dfrac{d}{dx} \ln(4x-3) = \dfrac{1}{4x-3} \dfrac{d}{dx}(4x-3) = \dfrac{4}{4x-3}$

Ex $\dfrac{d}{dx}(2^x) = \dfrac{d}{dx}\left((e^{\ln 2})^x\right) = \dfrac{d}{dx}\left(e^{x \ln 2}\right) = e^{x \ln 2} \cdot \ln 2 = 2^x \ln 2$

$2 = e^{\ln 2}$

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exam: 12 Q's   on Quest, exam from previous 408C   17 Q's

this exam is slightly more conceptual

Know $\sin^2\theta + \cos^2\theta = 1$

Know derivatives of trig f's

$$\lim_{x\to\infty} \frac{x^4+7x}{9x^4+3} = \frac{1}{9}$$

$$\lim_{x\to 0} \frac{1}{x\sin(x)} = \infty$$

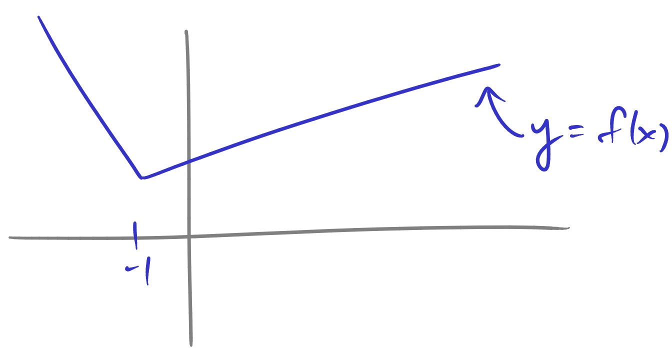
$$\left[\lim_{x\to 0} \frac{\sin x}{x} = 1 \quad \text{ie for small } x, \ \sin x \approx x\right]$$

$$\lim_{x\to 2^-} |x+3| = 5$$

$$|x+3| = \begin{cases} x+3 & x > -3 \\ -x-3 & x \le -3 \end{cases}$$

$$\lim_{x\to 2} |x+3| = 5$$

$$\lim_{x\to -3} |x+3| = 0$$



$$f(x) = \begin{cases} 2 & x = 0 \\ x+1 & x \neq 0 \end{cases}$$

$$\lim_{x\to 0} f(x) = 1$$