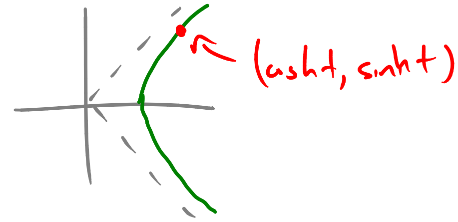
Last time: hyperbolic trig functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \text{etc.}$$



$$\cosh^2 x - \sinh^2 x = 1$$

[Rk The similarity to sin, cos comes from the complex-exp formula

$$e^{ix} = \sin x + i\cos x \longrightarrow \sin x = \frac{e^{ix} - e^{-ix}}{2i} \qquad \cos x = \frac{e^{ix} + e^{-ix}}{2}$$]

Derivatives of hyperbolic trig functions:

$$\frac{d}{dx}(\sinh x) = \cosh x$$

[why? $\frac{d}{dx}(\sinh x) = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) = \frac{e^x - (-e^{-x})}{2} = \frac{e^x + e^{-x}}{2} = \cosh x$]

$$\frac{d}{dx}(\cosh x) = \sinh x$$

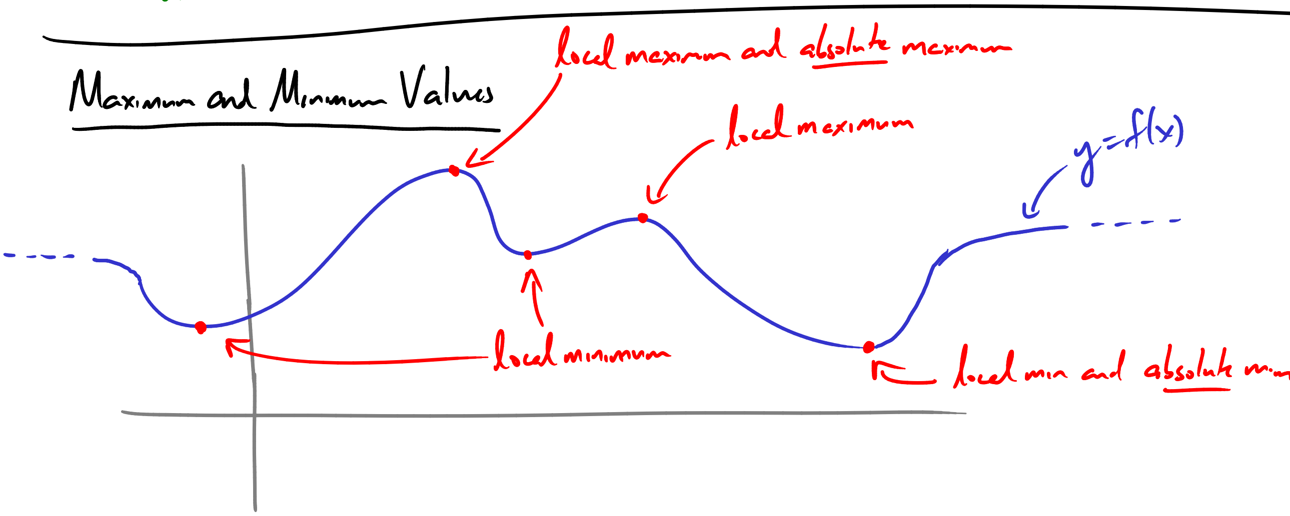
$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

Maximum and Minimum Values

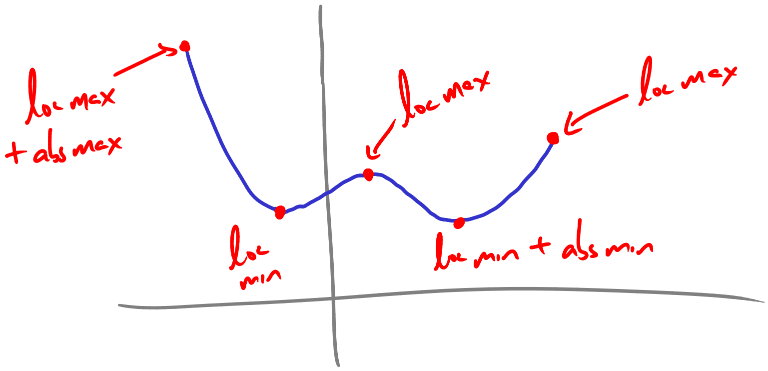


Say: $f(c)$ is <u>absolute max</u> value of f if $f(c) \geq f(x)$ for every x in domain of f.

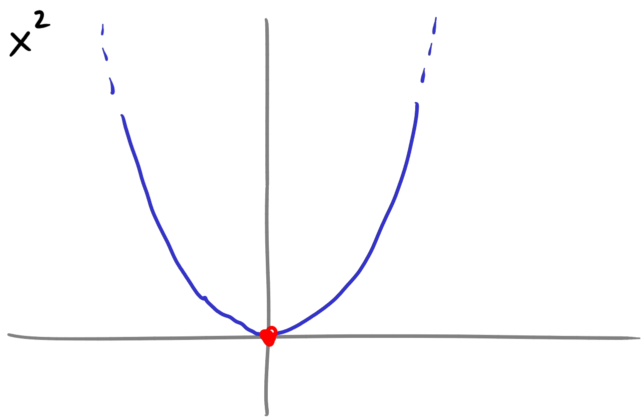
$f(c)$ is <u>absolute min</u> " " " " $f(c) \leq f(x)$ " " " " " " "

$f(c)$ is <u>local max</u> " " " " $f(c) \geq f(x)$ for every x near enough to c.

$f(c)$ is <u>local min</u> " " " " $f(c) \leq f(x)$ " " " " " " .



<u>Ex</u> $f(x) = x^2$



domain $= (-\infty, \infty)$

<u>no</u> absolute max or local max

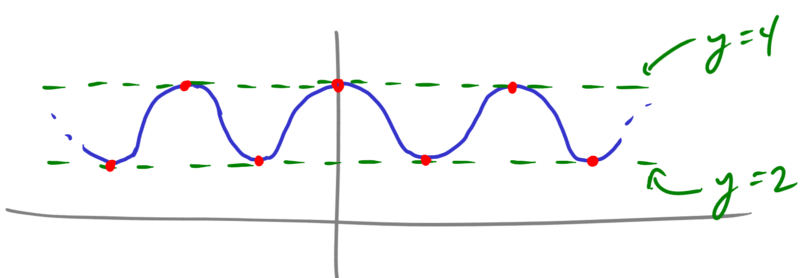
absolute minimum: $x=0$ $f(x)=0$

local minimum: $x=0$ $f(x)=0$

<u>Ex</u> $f(x) = 3 + \cos x$

(local or) absolute max: $f(x) = 4$ at $x = 0, 2\pi, -2\pi, 4\pi, -4\pi, \ldots$

(local or) absolute min: $f(x) = 2$ at $x = \pi, -\pi, 3\pi, -3\pi, 5\pi, -5\pi, \ldots$



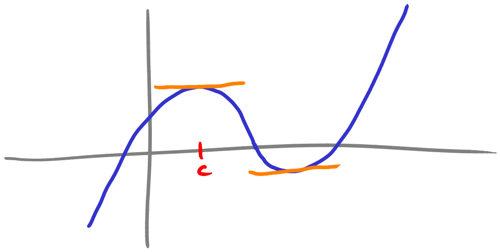
no other local max/min

<u>Fact</u>: <u>If</u> $f(x)$ has a local max or min at $x=c$,

<u>and</u> $f'(c)$ exists,

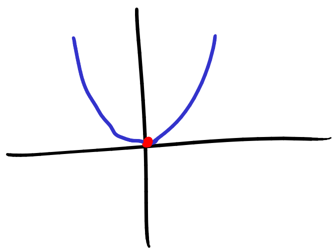
<u>and</u> the domain of f contains an interval around c (ie c isn't an endpoint of the domain)

Thm, $f'(c) = 0$.



(ie: places where the graph of $y=f(x)$ turns around and is differentiable are places where it has a horiz tangent)

Ex $f(x) = x^2$



$f'(x) = 2x$

So $f'(x) = 0$ only at $x = 0$

and $f'(x)$ exists for all x.

So the only possible place for a local max/min is $x = 0$.

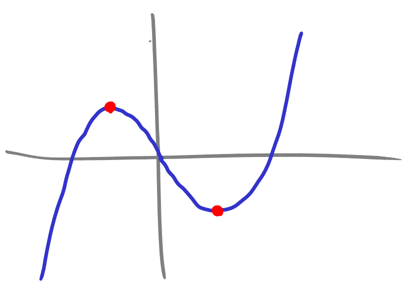
Indeed there is a local min at $x=0$. ✓

Ex $f(x) = x^3 - 3x$ $\quad$ $f'(x) = 3x^2 - 3$ $\quad$ exists for all x

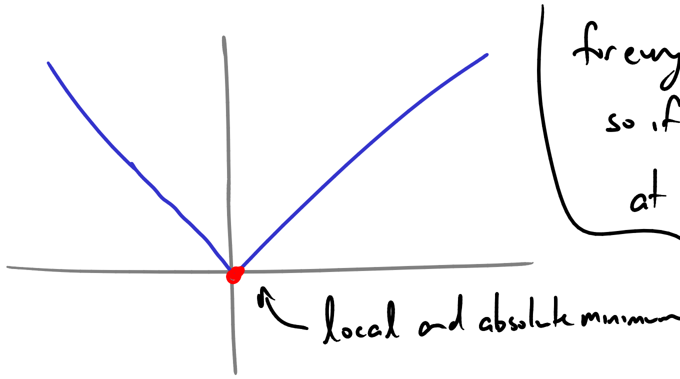
$f'(x) = 0$ only at $\quad 0 = 3x^2 - 3$

$0 = 3(x+1)(x-1)$ $\quad$ ie $x = 1$ or $x = -1$.

So the only possible places for loc min/max are $x=1$ and $x=-1$.



Ex $f(x) = |x|$



for every $x \neq 0$, $f'(x)$ exists, is $\neq 0$

so if $x \neq 0$, x is not a loc min/max.

at $x=0$, $f'(0)$ DNE.

So $x=0$ could be a loc min/max. (it is)

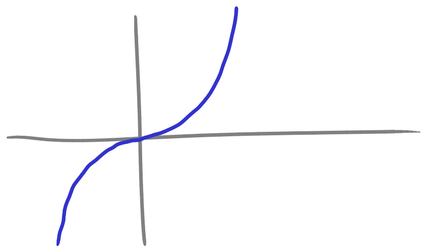
Ex $f(x) = x^3$

$f'(x) = 3x^2$ exists for all x, so only possible loc min/max will be where $f'(x) = 0$.

$f'(x) = 0$

$3x^2 = 0$ So the only possible loc min/max is at $x = 0$.

$x = 0$



But actually $x=0$ is not a loc min/max.

So this $f(x)$ has no local min/max.