Last time: hyperbolic trig functions

\[
\sinh x = \frac{e^x - e^{-x}}{2} \quad \cosh x = \frac{e^x + e^{-x}}{2} \quad \text{etc.}
\]

\[
\cosh^2 x - \sinh^2 x = 1
\]

\[
\begin{bmatrix}
\text{Re:} & \text{The similarity to } \sin, \cos \text{ comes from the complex-exp formula } \\
&e^{ix} = \sin x + i \cos x & \rightarrow & \sin x = \frac{e^{ix} - e^{-ix}}{2i} & \cos x = \frac{e^{ix} + e^{-ix}}{2}
\end{bmatrix}
\]

Derivatives of hyperbolic trig functions:

\[
\frac{d}{dx}(\sinh x) = \cosh x
\]

\[
\frac{d}{dx}(\cosh x) = \sinh x
\]

\[
\frac{d}{dx}(\tanh x) = \sech^2 x
\]

\[
\frac{d}{dx}(\coth x) = -\csch x \coth x
\]

\[
\frac{d}{dx}(\csch x) = -\sech x \tanh x
\]

\[
\frac{d}{dx}(\sech x) = -\sech^2 x
\]

Maximum and Minimum Values

local maximum and absolute maximum

local maximum

y = f(x)

local minimum

local minimum and absolute minimum
Say:

- $f(c)$ is absolute max value of $f$ if $f(c) \geq f(x)$ for every $x$ in domain of $f$.
- $f(c)$ is absolute min if $f(c) \leq f(x)$.
- $f(c)$ is local max if $f(c) > f(x)$ for every $x$ near enough to $c$.
- $f(c)$ is local min if $f(c) < f(x)$.

**Example**

$f(x) = x^2$

- Domain: $(-\infty, \infty)$
- No absolute max or local max
- Absolute minimum: $x=0$ $f(x) = 0$
- Local minimum: $x=0$ $f(x) = 0$

**Example**

$f(x) = 3 + \cos(x)$

- Local absolute max: $f(x) = 4$ at $x = 0, 2\pi, -2\pi, 4\pi, -4\pi, ...$
- Local absolute min: $f(x) = 2$ at $x = \pi, -\pi, 3\pi, -3\pi, 5\pi, -5\pi, ...$

- No other local max/min

**Fact:** If $f(x)$ has a local max or min at $x = c$, and $f(c)$ exists, and the domain of $f$ contains an interval around $c$ (i.e., $c$ is not an endpoint of the domain), then $f(c)$ is an absolute max or min.
Then, \( f'(c) = 0 \).

(ie: places where the graph of \( y=f(x) \) turns around and is differentiable are places where it has a horizontal tangent.)

**Ex** \( f(x) = x^2 \)

\[ f'(x) = 2x \]

So \( f'(x) = 0 \) only at \( x=0 \) and \( f'(x) \) exists for all \( x \).

So the only possible place for a local max/min is \( x=0 \).

Indeed there is a local min at \( x=0 \).

**Ex** \( f(x) = x^3 - 3x \)

\[ f'(x) = 3x^2 - 3 \]

exists for all \( x \)

\[ f(x) = 0 \] only at

\[ 0 = 3x^2 - 3 \]

\[ 0 = 3(x+1)(x-1) \] \( \text{i.e.} \) \( x=1 \) or \( x=-1 \).

So the only possible places for local max/min are \( x=1 \) and \( x=-1 \).

**Ex** \( f(x) = |x| \)

\[ \text{for every } x \neq 0, \quad f'(x) \text{ exists, is } 0 \]

so if \( x \neq 0 \), \( x \) is not a local min/max.

\[ \text{at } x=0, \quad f'(0) \text{ DNE.} \]

So \( x=0 \) could be a local min/max. (it is)

local and absolute minimum
Ex: \( f(x) = x^3 \)

\[ f'(x) = 3x^2 \] exists for all \( x \), so only possible local min/max will be when \( f'(x) = 0 \).

\[ f'(x) = 0 \]

\[ 3x^2 = 0 \]

\[ x = 0 \]

So the only possible local min/max is at \( x = 0 \).

But actually \( x = 0 \) is not a local min/max.

So this \( f(x) \) has no local min/max.