Lecture 27

HW10 due 3am Nov 9 (Friday)
HW11 will be 3am Nov 13 (Tue) as usual

Midterm 2 grades posted today or tomorrow.
Class average ≈ 86\%.

Good!
A train accelerates with constant acceleration, \( a(t) = 4 \text{ ft/s}^2 \).

At time \( t = 0 \) it has velocity 100 ft/s.

How far does it go in 20 s?

**Know:**
- \( a(t) = 4 \)
- \( v(0) = 100 \)
- \( s(0) = 0 \)

**Want:** \( s(20) \).

\( d \) is antideriv of \( v(t) \)
\( v(t) \) is antideriv of \( a(t) \)

\( \implies v(t) = 4t + C \)

and \( v(0) = 100 \), so \( 4(0) + C = 100 \)

\( C = 100 \)

so \( v(t) = 4t + 100 \)

then \( s(t) = 2t^2 + 100t + D \)

and \( s(0) = 0 \), so \( 2(0)^2 + 100(0) + D = 0 \)

\( D = 0 \)

so \( s(t) = 2t^2 + 100t \)

\( s(20) = 2(20)^2 + 100(20) = 800 + 2000 = 2800 \text{ ft} \)

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**Remake**

Every continuous function has an antiderivative.

But, e.g. the antiderivative of \( f(x) = e^{-x^2} \) cannot be written in terms of "elementary" functions (\( +, -, \div, \exp, \log, \ln, \sin, \cos, \tan, \arcsin, ... \)).

We give this antideriv a name, "error function" \( \text{erf}(x) \).
Areas

We all know areas of simple shapes:

\[
\text{rectangle} \quad A = \omega \cdot h
\]

How about more complicated shapes?

Let's try to calculate the area under the graph of \( y = f(x) \), over the x-axis, between \( x = a \) and \( x = b \).

Example: \( y = x^2 \).

Estimate the area of the region between \( y = f(x) \) and the x-axis and between \( x = 0 \) and \( x = 1 \).

Idea: approximate our region by a bunch of rectangles.

Total area of rectangles:

\[
\text{total area of rectangles} = \frac{1}{4} \left[ \left( \frac{1}{4} \right)^2 + \left( \frac{2}{4} \right)^2 + \left( \frac{3}{4} \right)^2 + 1^2 \right]
\]

\[
= \frac{15}{32}
\]
This gives an overestimate of the area under \( y = x^2 \) from 0 to 1.
(Let the rectangles completely cover that area)

It is the "estimated area using \( 4 \) rectangles and using the right endpoints of the intervals as sample points."

So, call it \( R_4 \).

Then \( R_4 = \frac{15}{32} \).

Ex. Estimate the same area, using \( 5 \) rectangles and left endpoints as sample points.

Estimated area
\[
L_5 = \frac{1}{5} \left( 0^2 + \left( \frac{1}{5} \right)^2 + \left( \frac{2}{5} \right)^2 + \left( \frac{3}{5} \right)^2 + \left( \frac{4}{5} \right)^2 \right) = \frac{30}{125}
\]
This is an underestimate of the actual area.