Lecture 34

26 Nov 2018

Last time: substitution

Ex $\displaystyle\int \frac{1}{x \ln x}\,dx \qquad u = \ln x \qquad du = \frac{1}{x}\,dx$

$\displaystyle = \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln|\ln x| + C$

$\displaystyle\int_0^1 x^6 + x^{1/6}\,dx = \cdots = 1$

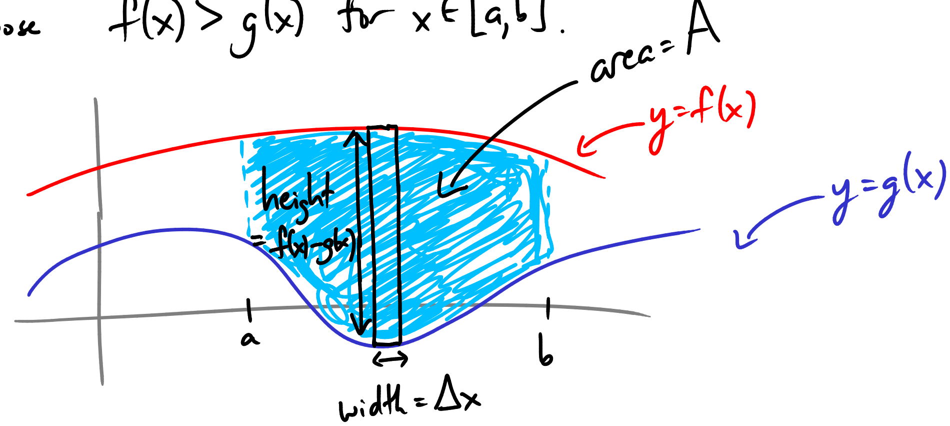
$\displaystyle\int_0^1 x^{13/2} + x^{2/13}\,dx = \cdots = 1$

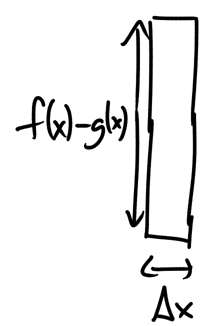
$\displaystyle\int_0^1 x^{p/q} + x^{q/p}\,dx = \cdots = 1$ Why?

Area between curves

Two curves $y = f(x)$ and $y = g(x)$.

Suppose $f(x) > g(x)$ for $x \in [a,b]$.

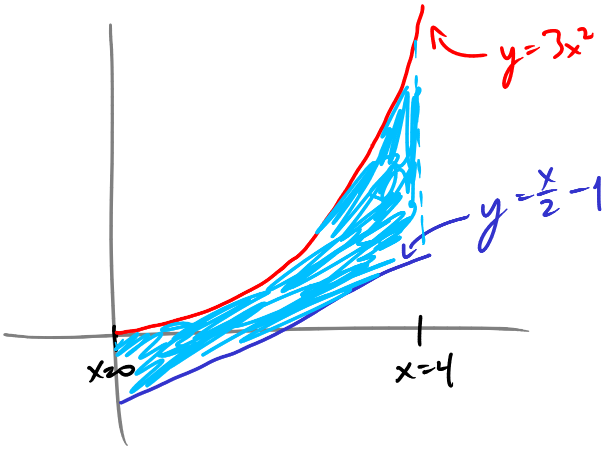




area of rectangle $= (f(x) - g(x))\,\Delta x$

total area $= A = \displaystyle\int_a^b (f(x) - g(x))\,dx$ (or: $\displaystyle\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$)

Ex Find the area between the curves $y = \underbrace{3x^2}_{f(x)}$, $y = \underbrace{\frac{x}{2} - 1}_{g(x)}$, $x = 0$, $x = 4$.



$$A=\int_0^4 f(x)-g(x)\,dx$$

$$=\int_0^4 3x^2-\left(\frac{x}{2}-1\right)dx$$

$$=\int_0^4 3x^2-\frac{x}{2}+1\,dx$$

$$=x^3-\frac{x^2}{4}+x\Big|_0^4$$

$$=(64-4+4)-0=\underline{\underline{64}}$$

$\underline{\text{Ex}}$ Find the area of the bounded region between the graphs $y=x^2$ and $y=2x-x^2$.

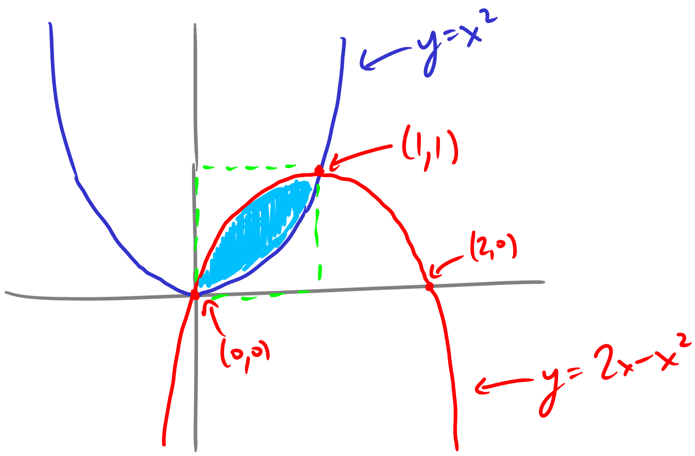
$y'=2-2x$

$y''=-2$

conc down, max at $x=1$

intercepts at

$0=y=x(2-x)$ $x=0,2$



$$A=\int_0^1 (2x-x^2)-(x^2)\,dx$$

$$=\int_0^1 2x-2x^2\,dx$$

$$=x^2-\frac{2}{3}x^3\Big|_0^1$$

$$=\left(1-\frac{2}{3}\right)-(0)=\underline{\underline{\frac{1}{3}}}$$

(to find intersection pts w/o drawing graphs: set $x^2=2x-x^2$

$2x^2-2x=0$

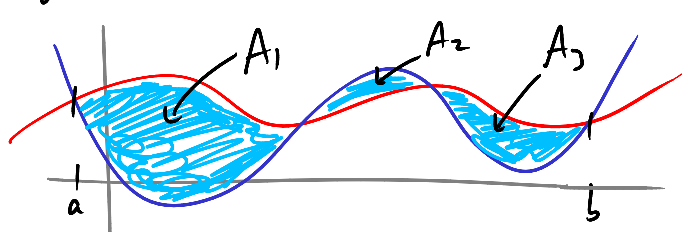
$2x(x-1)=0$

$x=0,1$)

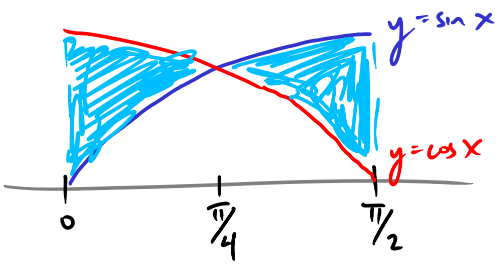
A rule that finds the area between $y=f(x)$ and $y=g(x)$ no matter which is bigger:

$$A=\int |f(x)-g(x)|\,dx$$

here $A=A_1+A_2+A_3$



Ex Find the area of the region between $y = \sin x$ and $y = \cos x$ for x ranging between $x = 0$ and $x = \frac{\pi}{2}$.



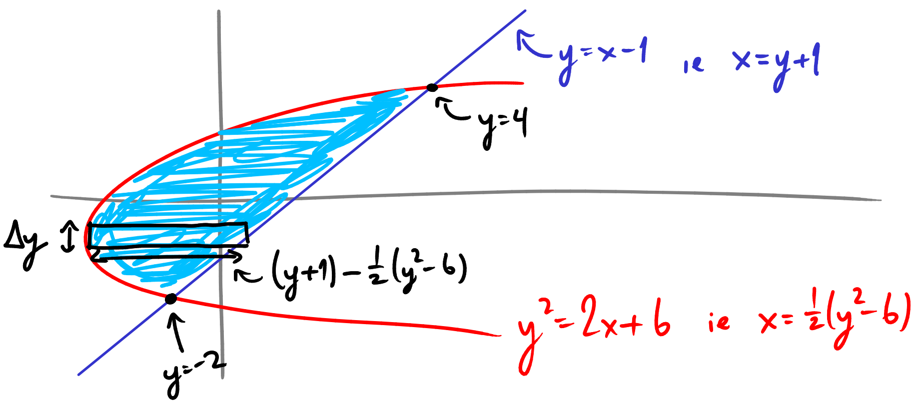
$$A = \int_0^{\pi/2} |\sin x - \cos x| \, dx$$

$$= \int_0^{\pi/4} \cos x - \sin x \, dx + \int_{\pi/4}^{\pi/2} \sin x - \cos x \, dx$$

$$= \ldots$$

$$= \underline{\underline{2(\sqrt{2}-1)}}$$

Ex Find the area between the parabola $y^2 = 2x + 6$ and the line $y = x - 1$.



intersections:

$$y + 1 = \frac{1}{2}(y^2 - 6)$$

$$2y + 2 = y^2 - 6$$

$$0 = y^2 - 2y - 8$$

$$0 = (y-4)(y+2)$$

$$y = -2, 4$$

$$\text{area} = \int_{-2}^{4} (y+1) - \frac{1}{2}(y^2 - 6) \, dy$$

$$= \int_{-2}^{4} -\frac{y^2}{2} + y + 4 \, dy$$

$$= \ldots = \underline{\underline{18}}$$

$$\int x^{7/2} + x^{2/7} \, dx = 1$$