Midterm 3 Dec 7 (next Friday)
in class, as usual

Last time: volumes

A common type of solid: "solid of revolution" — take some region in the x-y plane and revolve it around, say, the x-axis.

Ex

The cross section of such a solid is a circle of radius f(x).
So cross section area is \( \pi f(x)^2 \). We use this to get the volume

\[ A(x) \]
Ex Find the volume of the solid obtained by revolving the area between \( y = \sqrt{x} \) and the \( x \)-axis, around the \( x \)-axis, with \( x \) running from \( 0 \) to \( 2 \). 

\[
V = \int_0^2 dx \ A(x) = \int_0^2 dx \, \pi \cdot (\sqrt{x})^2 = \int_0^2 dx \, \pi \cdot x = \pi \cdot \left. \frac{x^2}{2} \right|_0^2 = 2\pi
\]

Ex same question for \( y = x \) (cone) 

\[
V = \int_0^2 dx \ A(x) = \int_0^2 dx \, \pi \cdot (x)^2 = \pi \cdot \left. \frac{x^3}{3} \right|_0^2 = \frac{8\pi}{3}
\]

Can also revolve around e.g. the \( y \)-axis.

Ex Find the vol of the solid obtained by revolving the region bounded by 

\[
x = y - y^2
\]

around the \( y \)-axis.

Slice by "horizontal" planes \( y = \) constant:

cross sections are circles of radius \( y - y^2 \)

\[
A(y) = \pi (y - y^2)^2
\]

\[
V = \int_0^1 dy \ A(y) = \int_0^1 dy \, \pi (y - y^2)^2
\]

\[
= \pi \int_0^1 dy \left( y^2 - 2y^3 + y^4 \right)
\]

\[
= \pi \left( \left. \frac{y^3}{3} - \frac{2y^4}{4} + \frac{y^5}{5} \right|_0^1 \right)
\]

\[
= \pi \cdot \left( \frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right) = \frac{\pi}{30}
\]

Another common shape: cross sections which are "washers".

Ex Let \( R \) be the region between \( y = \sqrt{x} \) and \( x = 2y \); 

Find the volume of the solid obtained by revolving \( R \) around the \( y \)-axis.
Slice by planes $y = \text{const.}$

Cross section:

Radius $= 2y$

\[
V = \int_{0}^{2} dy \ A(y)
\]

\[
= \pi \int_{0}^{2} dy \ (4y^2 - y^4)
\]

\[
= \cdots = \frac{64\pi}{15}
\]

**Ex.** Calculate the volume of a solid whose base is the region between $y = x, y = -x$ and $x=2$ and whose cross section at fixed $x$ are equilateral \( \Delta \)'s

\[
V = \int_{0}^{2} A(x) \ dx
\]

\[
A(x) = \text{area of equilateral } \Delta
\]

\[
w_{x} = 2x, \quad A(x) = \frac{\sqrt{3}}{4}x^2
\]

\[
V = \int_{0}^{2} A(x) \ dx = \int_{0}^{2} \frac{\sqrt{3}}{4}x^2 \ dx = \cdots = \frac{8\sqrt{3}}{3}
\]