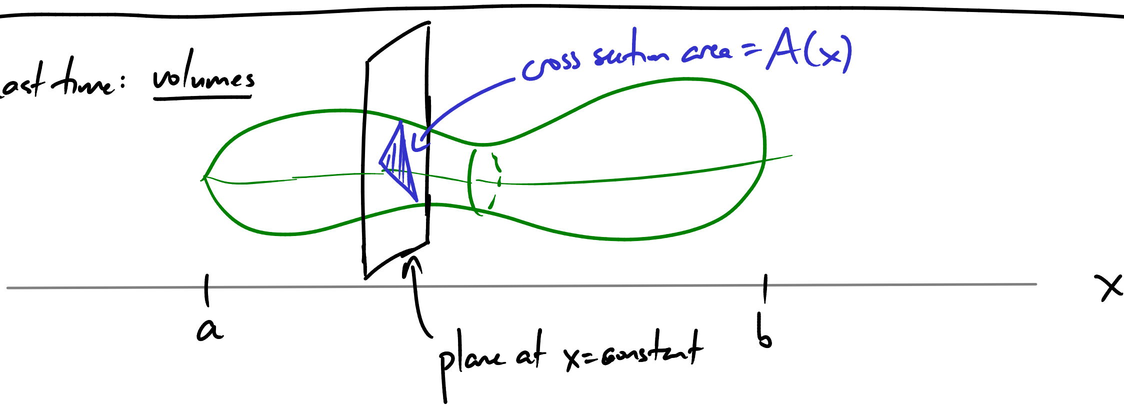
Lecture 36 | 30 Nov 2018

Midterm 3 Dec 7 (next Friday)

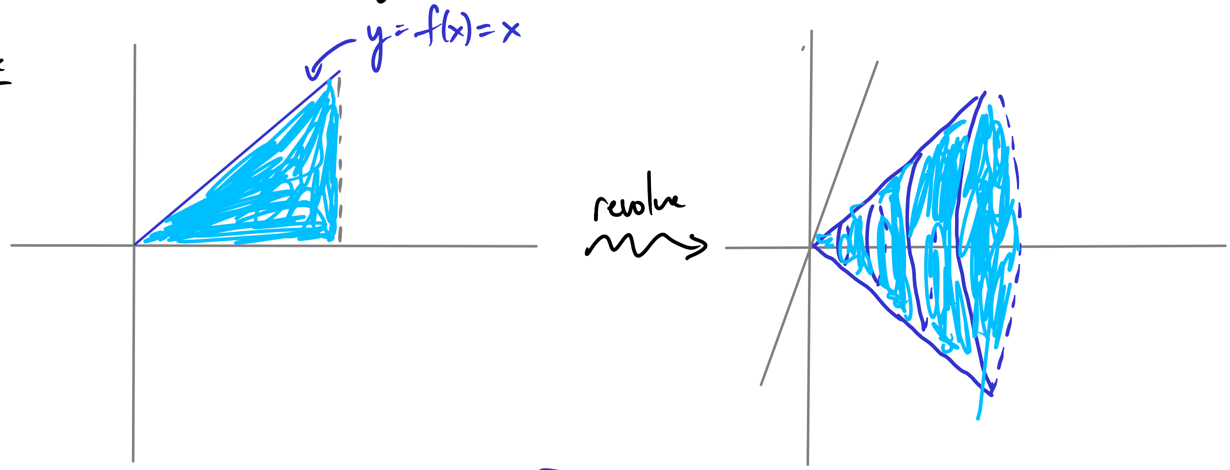
in class, as usual

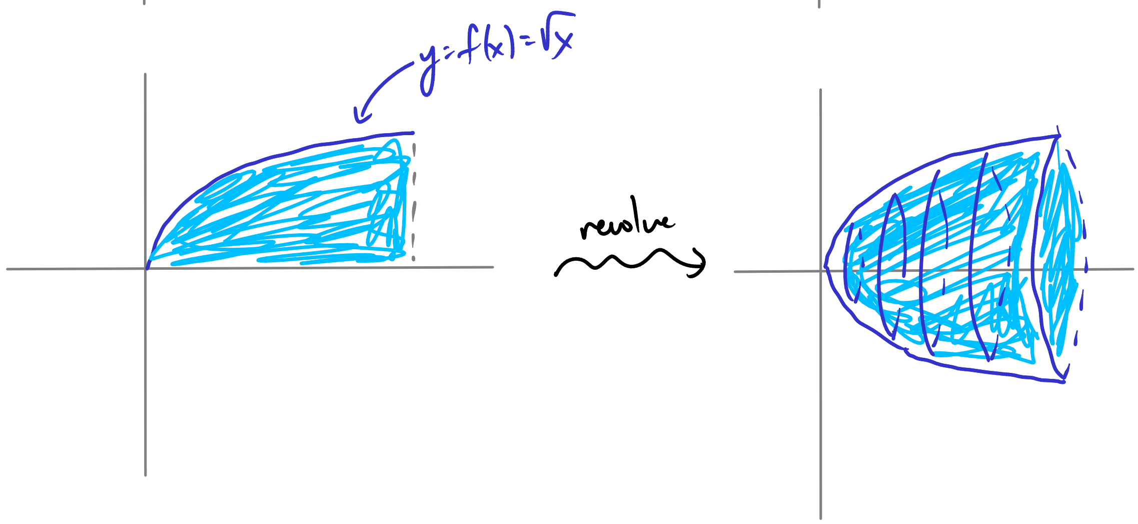
Last time: volumes



A common type of solid: "solid of revolution" — take some region in the x-y plane and revolve it around, say, the x-axis.

Ex





The cross section of such a solid is a circle of radius $f(x)$.

So cross section area is $\pi \cdot f(x)^2$. We use this to get the volume.

$$\underset{\displaystyle A(x)}{\overset{\parallel}{}}$$

A(x)

Ex Find the volume of the solid obtained by revolving the area between $y=\sqrt{x}$ and the x-axis, around the x-axis, with x running from 0 to 2. (bullet)

$$V = \int_0^2 dx\, A(x) = \int_0^2 dx\, \pi \cdot (\sqrt{x})^2 = \int_0^2 dx\, \pi \cdot x = \pi \cdot \frac{x^2}{2}\Big|_0^2 = \underline{\underline{2\pi}}$$

Ex same question for $y=x$ (cone)

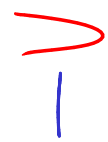
$$V = \int_0^2 dx\, A(x) = \int_0^2 dx\, \pi \cdot (x)^2 = \pi \frac{x^3}{3}\Big|_0^2 = \underline{\underline{\frac{8\pi}{3}}}$$

Can also revolve around e.g. the y-axis.

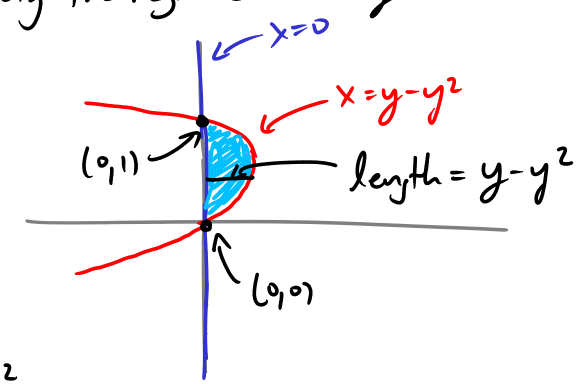
Ex Find the vol of the solid obtained by revolving the region bounded by

$x = y - y^2$

$x = 0$



around the y-axis.



slice by "horizontal" planes y = constant:

cross sections are circles of radius $= y - y^2$ $\quad A(y) = \pi (y-y^2)^2$

$$V = \int_0^1 dy\, A(y)$$

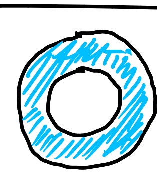
$$= \int_0^1 dy\, \pi (y-y^2)^2 = \pi \int_0^1 (y-y^2)^2\, dy$$

$$= \pi \int_0^1 y^2 - 2y^3 + y^4\, dy$$

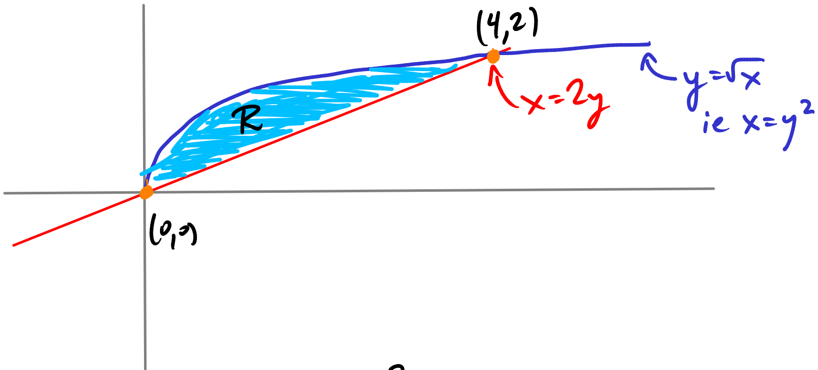
$$= \pi \left(\frac{y^3}{3} - \frac{2y^4}{4} + \frac{y^5}{5} \Big|_0^1 \right)$$

$$= \pi \cdot \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right) = \underline{\underline{\frac{\pi}{30}}}$$

Another common shape: cross sections which are "washers"



Ex Let R be the region between $y = \sqrt{x}$ and $x = 2y$.
Find the volume of the solid obtained by revolving R around the y-axis.



Slice by planes y = const.
Cross section:

$$V = \int_0^2 dy\, A(y)$$

$$= \pi \int_0^2 dy\; 4y^2 - y^4$$

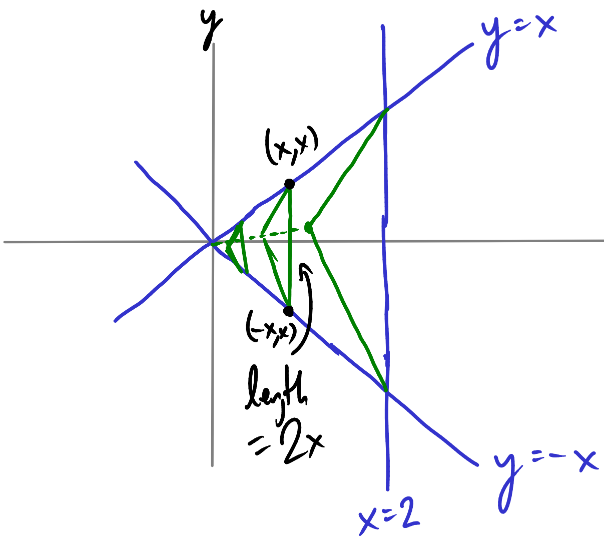
$$= \cdots = \frac{64\pi}{15}$$

$$A(y) = \pi(2y)^2 - \pi(y^2)^2$$

↑ area of big circle — ↑ area of small circle (hole)

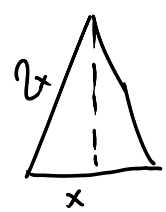
$$= \pi \cdot (4y^2 - y^4)$$

Ex Calculate the volume of a solid whose base is the region between $y=x$, $y=-x$ and $x=2$ and whose cross sections at fixed x are equilateral Δ's



$$V = \int_0^2 A(x)\,dx$$

$A(x)$ = area of equilat. Δ with side length = $2x$



$$A(x) = \sqrt{3}x^2$$

$$V = \int_0^2 A(x)\,dx = \int_0^2 \sqrt{3}\cdot x^2\,dx = \cdots = \frac{8}{3}\sqrt{3}$$