A definition from physics:
if an object moves for a distance $\Delta x$, 
acted on by a constant force $F$,
then we say that the force does work on the object,
$W = F \cdot \Delta x$

Ex: To lift a rock weighing 1 kg 
for a height $\Delta x = \frac{1}{2} m$ (with constant speed)
we have to exert a force $F = mg = (1 \text{ kg})(9.8 \text{ m/s}^2) = 9.8 \text{ kg} \cdot \text{m/s}^2$.
so the work we have to do is $W = F \cdot \Delta x = (9.8 \text{ kg} \cdot \text{m/s}^2) \cdot (\frac{1}{2} \text{ m})$
$= 4.9 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 4.9 \text{ J}$

What if the force is not constant?
Doesn't make sense to write $W = F \cdot \Delta x$

Instead, $W = \int F \, dx$
(One way to think about this: break the process up into many sub-processes, $W \approx \sum F(x_i^+) \Delta x \rightarrow W = \int F \, dx$)

Ex: A block is attached to a spring
When the block is at position $x$
the spring exerts a force $F = -kx$ ("Hooke's Law")
If \( k = 2 \frac{N}{m} \), what is the work done by the spring on the block as it moves from \( x = 0 \) to \( x = 0.03 \) m?

\[
W = \int_0^{0.03} F \, dx = \int_0^{0.03} (-2) \times dx = -x \bigg|_0^{0.03} = -0.0009 \, \text{J}
\]

Why do we want to calculate the work?

**Because:**

**Total work**

\[
W = \int_{x_0}^{x_1} F \, dx = \int_{x_0}^{x_1} ma \, dx = \int_{x_0}^{x_1} m \frac{du}{dt} \, dx = \int_{v_0}^{v_1} m \cdot v \, dv = \frac{1}{2} mv_1^2 - \frac{1}{2} mv_0^2 = \text{net change in } \frac{1}{2} mv^2 \rightarrow \text{kinetic energy}
\]

**Optimization**

What is the area of the largest rectangle we can inscribe between the graph \( y = 16 - x^4 \) and \( y = 0 \)?

\[
\text{Area} = 2x(16 - x^4)
\]

\[
16 - x^4 \bigg|_{x=2} = 0
\]

\[
2x
\]
So, maximize \( A = 2x(16-x^4) \) for \( x \in [0, 2] \)

critical point: \( \frac{dA}{dx} = 32 - 10x^4 = 0 \)

\[
32 = 10x^4 \\
\frac{16}{5} = x^4 \\
x = \sqrt[4]{\frac{16}{5}} = \frac{2}{\sqrt[4]{5}} \\
A = 32x - 2x^5
\]

\[
\int_0^{\frac{2}{\sqrt[4]{5}}} 2f(4x) \, dx = ?
\]

\[
\begin{align*}
& u = 4x \\
& du = 4 \, dx \\
& \frac{du}{4} = dx \\
& \int_{u=0}^{u=6} 2f(u) \cdot \frac{du}{4}
\end{align*}
\]

\[
= \frac{1}{2} \int_0^6 f(u) \, du = \frac{1}{2} (18) = 9
\]

\[
\int_0^6 f(x) \, dx = 18
\]

\[
\int_0^6 f(t) \, dt =
\]

\[
\int_0^1 x^2 \, dx = \frac{1}{3}
\]

\[
\int_0^1 t^2 \, dt = \frac{1}{3}
\]

\[
\int_0^1 u^2 \, du = \frac{1}{3}
\]

Calculate volume of an object whose cross-sections between \( x = 4 - y^2 \) and \( x = 0 \), where \( y = \sqrt[4]{4-x} \) and \( y = \sqrt{4-x} \), are squares.

\[
V = \int_0^4 A(x) \, dx
\]

\[
= \int_0^4 4 \cdot (4-x) \, dx
\]

\[
= -
\]

\[
\text{area} = 4 \cdot (4-x)
\]
\[ \sqrt[4]{75} \text{ by Newton's Method:} \quad \text{start with } \sqrt[4]{81} = 3 \]
\[ x^4 = 75 \quad x^4 - 75 = 0 \]

Look at the function \( f(x) = x^4 - 75 \) \quad \text{want to solve } f(x) = 0

Initial guess: \( x_0 = 3 \)

Next guess: \( x_1 = 3 - \frac{f(3)}{f'(3)} \)

Here \( f(3) = 81 - 75 = 6 \)
\( f'(x) = 4x^3 \)
\( f'(3) = 4 \cdot 3^3 = 108 \)

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

\[ f''(3) = \frac{6}{108} = \frac{1}{18} \]

\[ f'(x) = 4x^3 \]