Lecture 2

31 Aug 2012

Administration:

HW02 due Tuesday 3am
HW01 due Friday 3am

My office hours Friday 11–12, Monday 11–12 } RLM 9.134

TA office hours Monday 12:30–2, Wednesday 12:30–2 } RLM 13.152

Office hours Monday as usual, TA office hour Friday 12:30–2 RLM 13.152

QUEST fee \$25

Last time:

$$f(x) \rightsquigarrow g(x) = \int_a^x f(t)\,dt$$

FTC 1: $\frac{d}{dx} g(x) = f(x)$

Ex $\frac{d}{dx} \int_3^x \sin(1+t)\,dt = \sin(1+x)$

Ex What is $\frac{d}{dx}\left[\int_1^{x^2} \frac{1}{1+t}\,dt\right]$?

Not just $\frac{1}{1+x^2}$!

Use Chain Rule: let $u(x) = x^2$

Then $\frac{d}{dx}\int_1^{u(x)} \frac{1}{1+t}\,dt = \frac{du}{dx}\cdot\frac{d}{du}\int_1^u \frac{1}{1+t}\,dt$

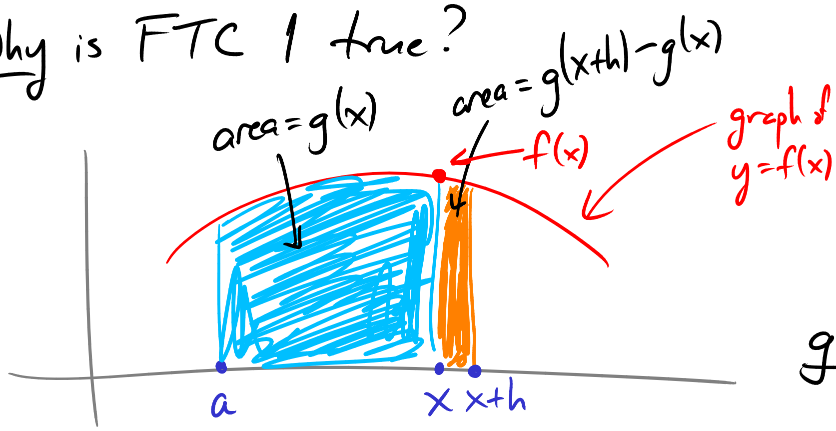
$$= \frac{d}{dx}(x^2) \cdot \frac{1}{1+u}$$

$$= 2x \cdot \frac{1}{1+x^2} = \frac{2x}{1+x^2}$$

__Ex__ What is $\frac{d}{dx} \int_x^5 \tan(t^3)\,dt$

$$= \frac{d}{dx}\left(-\int_5^x \tan(t^3)\,dt\right) = \underline{\underline{-\tan(x^3)}}.$$

Why is FTC 1 true?



To study __derivative__ of $g(x)$:

$$\frac{g(x+h)-g(x)}{h} \approx \frac{h \cdot f(x)}{h} = f(x)$$

So, $$g'(x) = \lim_{h \to 0} \frac{g(x+h)-g(x)}{h} = f(x)$$

__FTC 2__ Suppose $f(x)$ is a continuous function on $[a,b]$ and $F(x)$ is an __antiderivative__ of $f(x)$,

i.e. $$\frac{d}{dx}F(x) = f(x).$$

__Then__, $$\int_a^b f(x)\,dx = F(b) - F(a).$$

Ex $\int_2^4 x^3\,dx = ?$

x^3 ← $f(x)$

Pick $F(x) = \frac{1}{4}x^4$. $\frac{d}{dx}F(x) = \frac{1}{4}\cdot 4x^3 = x^3 = f(x)$

so $F(x)$ is an antideriv. of $f(x)$.

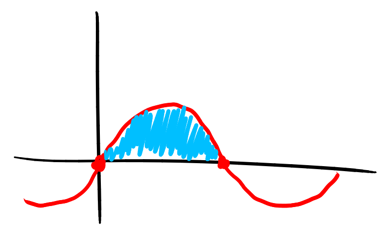
So $\int_2^4 x^3\,dx = F(4) - F(2) = \frac{1}{4}\cdot 4^4 - \frac{1}{4}\cdot 2^4 = 64 - 4 = \underline{\underline{60}}$

Another notation: $\int_2^4 x^3\,dx = \frac{x^4}{4}\Big]_2^4 = \frac{1}{4}\cdot 4^4 - \frac{1}{4}\cdot 2^4 = 64 - 4 = \underline{\underline{60}}$

Ex $\int_0^{\pi} \sin x\,dx = -\cos x\Big]_0^{\pi}$

$= \left(-(-1)\right) - (-1)$

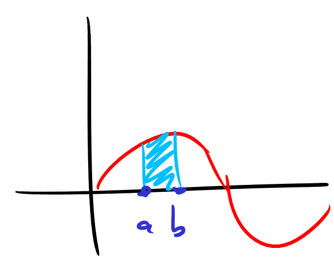
$= \underline{\underline{2}}$



$\int_a^b \sin x\,dx = -\cos x\Big]_a^b$

$= -\cos b - (-\cos a)$

$= \underline{\underline{\cos a - \cos b}}$



Some useful antiderivatives

$f(x)$	$F(x)$
$x^n \quad (n \neq -1)$	$\frac{1}{n+1} x^{n+1}$
$x^{-1} \quad (x>0)$	$\ln(x)$
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
e^x	e^x
$\sec^2(x)$	$\tan(x)$

Ex

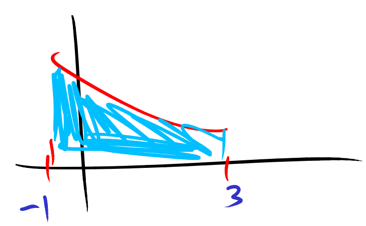
$$\int_{-1}^{3} e^{-2x}\, dx = \left.\frac{e^{-2x}}{-2}\right]_{-1}^{3}$$

because, by chain rule,

$$\frac{d}{dx}\left(e^{-2x}\right) = \frac{d}{dx}(-2x)\, e^{-2x} = -2e^{-2x}$$

$$= \frac{e^{-6}}{-2} - \frac{e^{2}}{-2}$$

$$= -\frac{1}{2}\left(e^{-6} - e^{2}\right) = \frac{1}{2}\left(e^{2} - e^{-6}\right)$$



Ex

$$\int_4^9 \sqrt{x}\, dx = \int_4^9 x^{1/2}\, dx = \left.\frac{x^{3/2}}{3/2}\right]_4^9 = \left.\frac{2}{3} x^{3/2}\right]_4^9$$

$$= \frac{2}{3}\left(9^{3/2} - 4^{3/2}\right)$$

but $9^{3/2} = \left(9^{1/2}\right)^3 = 3^3 = 27$, $4^{3/2} = \left(4^{1/2}\right)^3 = 2^3 = 8$

$$= \frac{2}{3}(27-8) = \frac{2}{3}(19) = \frac{38}{3}$$

Ex $\int_6^{-2} \frac{1}{x+6}\,dx = \int_6^{-2} (x+6)^{-1}\,dx$

$= \ln(x+6)\Big]_6^{-2}$

[Chain rule: $\frac{d}{dx}\ln(x+6) = \frac{d}{dx}(x+6)\cdot\frac{1}{x+6}$]

$= \ln(4) - \ln(12)$

$= \ln\left(\frac{4}{12}\right)$

$= \ln\left(\frac{1}{3}\right)$

$= \underline{\underline{-\ln 3}}$

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