Lecture 3 5 Sep 2012

Housekeeping:

HW02 due yesterday
HW01 due Friday 3am
HW03 due Tuesday 3am
HW04 due next Tue 3am
⋮

Office hours:

Javier today 12:30–2:00

This week, I'm gone Th–Tu — will have office hours this Wed and next Wed at 5pm, instead of the usual time.

Last time: FTC I, II

I: $\int_a^x f(x)\,dx$ is an antiderivative of $f(x)$

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x).$$

II: $\int_a^b f(x)\,dx = F(b) - F(a) = F\Big|_a^b$

where $F(x)$ is any antideriv of $f(x)$.

Indefinite Integrals (Ch 5.4)

Notation: $\int f(x)\,dx$ means the general antiderivative of $f(x)$.

Ex $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$ [C = arbitrary constant]

$\int \sin x \, dx = -\cos x + C$ $\int \cos x \, dx = \sin x + C$

$\int \sec^2 x \, dx = \tan x + C$ $\int \csc^2 x \, dx = -\cot x + C$

$\int \sec x \tan x \, dx = \sec x + C$ $\int \csc x \cot x \, dx = -\csc x + C$

[see table in text p. 398]

Ex Find $\int 10x^4 + 6\sec^2 x \, dx.$

$= 10 \cdot \frac{x^5}{5} + 6 \cdot \tan x + C = 2x^5 + 6\tan x + C$

(or: $= 10 \cdot \left(\frac{x^5}{5} + C\right) + 6 \cdot (\tan x + C)$;
but we just collect all the C's into a single C at the end)

Ex Find $\int_0^{\pi/4} 10x^4 + 6\sec^2 x \, dx$

$= 2x^5 + 6\tan x + C \Big|_0^{\pi/4}$

$= \left(2\left(\frac{\pi}{4}\right)^5 + 6 \cdot 1 + C\right) - (0 + 0 + C)$

$= 2\left(\frac{\pi}{4}\right)^5 + 6 = \frac{\pi^5}{512} + 6$

Ex Find $\int u^{2/3}\,du.$ $\quad n = 2/3 \quad n+1 = 5/3$

$$= \underline{\underline{\frac{3}{5}u^{5/3} + C}}$$

Ex Find a function $F(u)$ with $\frac{dF}{du} = u^{2/3}$ $\quad (*)$

and $F(1) = 1$.

$(*)$ says F is an indefinite integral of $u^{2/3}$.

i.e. $F(u) = \int u^{2/3}\,du = \frac{3}{5}u^{5/3} + C.$

And $F(1) = 1$ means $\frac{3}{5}(1^{5/3}) + C = 1$

$$\frac{3}{5} + C = 1$$

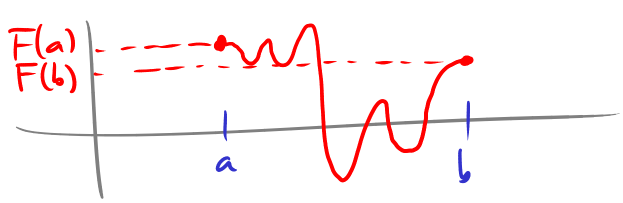
$$C = 2/5$$

So $\underline{\underline{F(u) = \frac{3}{5}u^{5/3} + \frac{2}{5}}}$

<u>Net change</u> (Ch 5.4)

Suppose have function $F(t)$. $\quad \frac{dF}{dt} = F'(t) =$ the rate of change of F.

$$\int_a^b F'(t)\,dt = F(b) - F(a) = \text{net change of } F(t) \text{ as } t \text{ goes from } a \text{ to } b.$$



Ex Water flows into a reservoir at the rate $(10t+6)\ ft^3/s$

(t measured in seconds)

The reservoir contains $400\ ft^3$ of water at time $t=0$.

How much does it contain at time $t=10$?

The net change from $t=0$ to $t=10$ is

$$\int_0^{10} (10t+6)\,dt = 5t^2+6t \Big|_0^{10}$$

$$= \left(5(10^2)+6(10)\right) - \left(5(0)^2+6(0)\right)$$

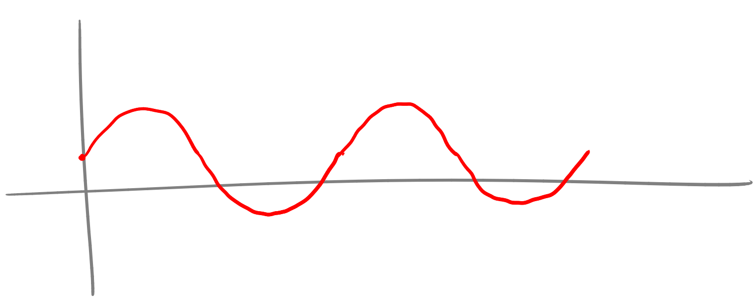
$$= 560-0 = 560\ ft^3$$

So the amount of water at $t=10$

is $400+560 = \underline{\underline{960\ ft^3}}$

Ex A rechargeable battery is connected to a load that can charge or discharge it.

The current flowing into the battery is $\sin(\pi t)+\frac{1}{2}$



If the battery starts with 10 units of charge at $t=0$ how much charge does it have at time $t=6$?

$Q(t)$ = charge at time t

$Q(0) = 10$

$$Q(6) - Q(0) = \int_0^6 Q'(t)\,dt = \int_0^6 \sin(\pi t) + \tfrac{1}{2}\,dt$$

$$= -\frac{1}{\pi}\cos(\pi t) + \frac{t}{2}\Big]_0^6$$

$$\left(\frac{d}{dt}\left(-\frac{1}{\pi}\cos \pi t\right) = -\frac{1}{\pi}(-\sin \pi t)\cdot \pi\right)$$

$$= \left(-\frac{1}{\pi}(1) + 3\right) - \left(-\frac{1}{\pi} + 0\right)$$

$$= 3$$

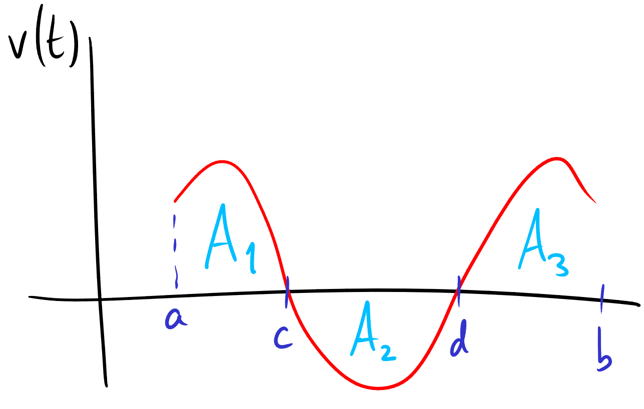
so $\quad Q(6) = 10 + 3 = \underline{\underline{13}}$

Total displacement

Remember: if $s(t)$ = position [along some line]

$s'(t) = v(t)$ velocity

[$v(t) > 0$: $s(t)$ increasing
i.e. moving to the right
$v(t) < 0$: $s(t)$ decreasing
i.e. moving to the left]



Total displacement $\quad s(b) - s(a) = \int_a^b v(t)\,dt = A_1 + A_3 - A_2$

Total distance $\quad A_1 + A_2 + A_3 = \int_a^b |v(t)|\,dt$

$$= \int_a^c v(t)\,dt + \int_c^d (-v(t))\,dt + \int_d^b v(t)\,dt$$

Ex A particle moves along a line with $v(t) = t^2 - t - 6$ m/s. (t in sec) from time $t=1$ to $t=4$.

a) What is the total displacement?

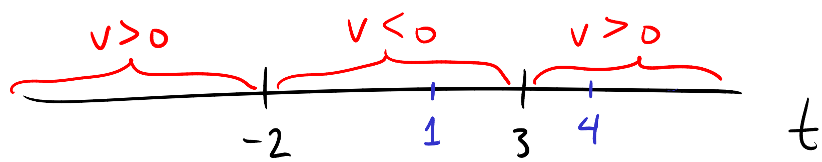
$$\Delta s = s(4) - s(1) = \int_1^4 v(t) = \int_1^4 t^2 - t - 6\,dt$$

$$= \cdots = -\frac{9}{2}\,m$$

(i.e. $\frac{9}{2}$m to the left)

b) What is the total distance?

$$\int_1^4 |v(t)|\,dt \qquad\qquad v(t) = (t-3)(t+2)$$



So:

$$\int_1^4 |v(t)|\,dt = \int_1^3 -v(t)\,dt + \int_3^4 v(t)\,dt$$

$$= \int_1^3 -t^2+t+6\,dt + \int_3^4 t^2-t-6\,dt$$

$$= -\tfrac{1}{3}t^3 + \tfrac{1}{2}t^2 + 6t\Big]_1^3 + \tfrac{1}{3}t^3 - \tfrac{1}{2}t^2 - 6t\Big]_3^4$$

$$= \tfrac{22}{3} + \tfrac{17}{6} = \underline{\underline{\tfrac{61}{6}}}$$