Lecture 8

19 Sep 2012

Extra examples of volume problems at

http://www.ma.utexas.edu/users/neitzke

Integration By Parts (Ch 7.1)

Product rule:

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)\,g(x) + f(x)\,g'(x)$$

Take $\int dx$ of both sides:

$$f(x)\,g(x) = \int f'(x)\,g(x)\,dx + \int f(x)\,g'(x)\,dx$$

$$\Longrightarrow \int f(x)\,g'(x)\,dx = f(x)g(x) - \int f'(x)\,g(x)\,dx$$

Or: write

$$u = f(x) \qquad v = g(x)$$
$$du = f'(x)\,dx \qquad dv = g'(x)\,dx$$

$$\Longrightarrow \int u\,dv = uv - \int v\,du$$

Ex Find $\int x \sin(x)\,dx$

Say

$$u = x \qquad v = -\cos(x)$$
$$du = dx \qquad dv = \sin(x)\,dx$$

Then $\int x\sin(x)\,dx = uv - \int v\,du$

$$= x(-\cos x) - \int (-\cos x)\, dx$$
$$= -x\cos x + \int \cos x\, dx$$
$$= -x\cos x + \sin x + C$$

<u>Ex</u> Find $\int \ln x\, dx$.

$$u = \ln x \qquad v = x$$
$$du = \frac{1}{x}\, dx \qquad dv = dx$$

$$\int \ln x\, dx = \int u\, dv = uv - \int v\, du$$
$$= x\ln x - \int x \cdot \frac{1}{x}\, dx$$
$$= x\ln x - \int 1\, dx$$
$$= x\ln x - x + C$$
$$= x(\ln x - 1) + C$$

<u>Ex</u> Find $\int e^t t^2\, dt$.

[Suppose we try $u = e^t \qquad v = \frac{1}{3}t^3$
$du = e^t dt \qquad dv = t^2 dt$

Then $\int e^t t^2 dt = \int u\, dv = uv - \int v\, du = \frac{1}{3}t^3 e^t - \int \frac{1}{3}t^3 e^t\, dt$]

Take $u = t^2 \qquad v = e^t$
$du = 2t\, dt \qquad dv = e^t dt$

$$\int e^t t^2 dt = \int u\, dv = uv - \int v\, du = t^2 e^t - \int e^t\, 2t\, dt$$

Use $\int$ by parts again: <u>new</u> u, v

$$u = 2t \qquad v = e^t$$
$$du = 2\,dt \qquad dv = e^t\,dt$$

$$= t^2e^t - \int u\,dv$$

$$= t^2e^t - \left(uv - \int v\,du\right)$$

$$= t^2e^t - 2te^t + \int 2e^t\,dt$$

$$= t^2e^t - 2te^t + 2e^t + C$$

$$= e^t(t^2 - 2t + 2) + C$$

Ex $\int_0^{\pi} t \sin(3t)\,dt$

$$u = t \qquad v = -\tfrac{1}{3}\cos(3t)$$
$$du = dt \qquad dv = \sin(3t)\,dt$$

$$\int_0^{\pi} t\sin(3t)\,dt = \int_{t=0}^{t=\pi} u\,dv = uv\Big]_0^{\pi} - \int_0^{\pi} v\,du$$

$$= (t)\left(-\tfrac{1}{3}\cos(3t)\right)\Big]_0^{\pi} - \int_0^{\pi}\left(-\tfrac{1}{3}\cos(3t)\right)dt$$

$$= -\tfrac{1}{3}t\cos(3t)\Big]_0^{\pi} + \tfrac{1}{3}\left(\tfrac{1}{3}\sin(3t)\right)\Big]_0^{\pi}$$

$$= \left(-\tfrac{\pi}{3}\cos 3\pi - 0\right) + \tfrac{1}{9}\left(\sin 3\pi - \sin 0\right)$$

$$= \tfrac{\pi}{3} - 0 + \tfrac{1}{9}(0 - 0) = \underline{\underline{\tfrac{\pi}{3}}}$$

Ex $\int e^x \sin x \, dx$

Try $u = e^x$ $\quad v = -\cos x$

$du = e^x dx \quad dv = \sin x \, dx$

$$\int e^x \sin x \, dx = \int u \, dv = uv - \int v \, du$$

$$= -e^x \cos x - \int (-\cos x) e^x \, dx$$

$$= -e^x \cos x + \int e^x \cos x \, dx$$

Int. by parts <u>again</u>:

$u = e^x \quad v = \sin x$

$du = e^x dx \quad dv = \cos x \, dx$

$$= -e^x \cos x + \int u \, dv$$

$$= -e^x \cos x + uv - \int v \, du$$

$$= -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$

So, we got

$$\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$

so $$2 \int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$$

$$\underline{\underline{\int e^x \sin x \, dx = \frac{1}{2} e^x (-\cos x + \sin x) + C}}$$