Lecture 16

8 Oct 2012

Exam tomorrow night 7pm   GSB 2.124

2 hours

17 multiple-choice Q's

<u>No calculators</u>

No penalty for guessing

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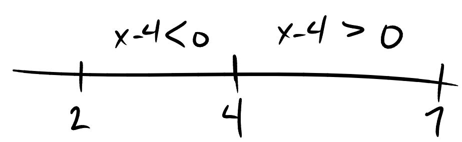
(1) Problems are in random order — first few are actually among <u>hardest</u> pbs.

(2) The $\int$'s will not look easy — some need 2 steps (e.g. u-sub, then $\int$ by parts)

Absolute values: $\displaystyle\int_2^7 |x-4|\,dx$



$$= \int_2^4 (-x+4)\,dx + \int_4^7 (x-4)\,dx$$

When to use trig sub? When you see $\sqrt{\pm x^2 \pm a^2}$ in the integrand and simpler methods haven't worked.

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C \qquad \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1}x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

$u = \cos x$
$du = -\sin x \, dx$

$$= \int -\frac{du}{u}$$

$$= -\ln|u| = -\ln|\cos x| = \ln\left|\frac{1}{\cos x}\right|$$

$$= \ln|\sec x|$$

$s(t) = \text{position}$

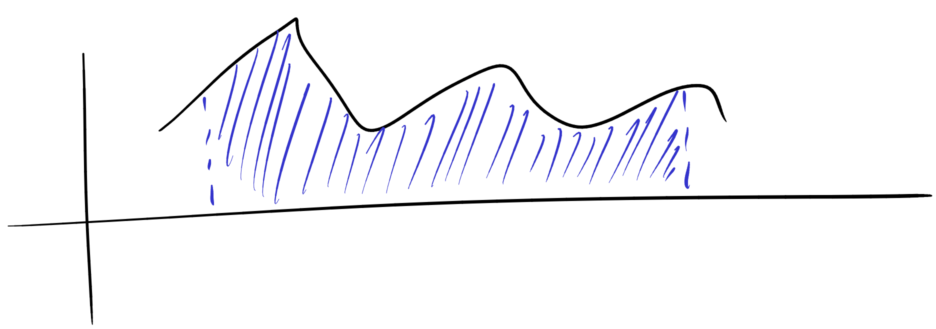
$\frac{ds}{dt} = \text{velocity}$

$\frac{d^2 s}{dt^2} = \text{accel.}$

$s(t) = \int v(t) \, dt = \text{position}$

$v(t) = \text{velocity}$

$v(t)$



Total distance traveled $= \int |v(t)| \, dt$

Total displacement $= \int v(t) \, dt$

$$\int \frac{1}{\sqrt{9+8x-x^2}}\,dx$$

complete the square: $9+8x-x^2$

$$= -\left(x^2-8x-9\right)$$

$$= -\left(\underbrace{(x-4)^2}_{x^2-8x+16}-25\right)$$

$$= 25-(x-4)^2$$

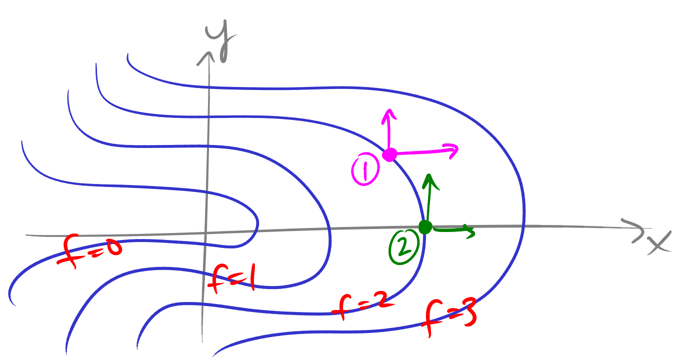
$$\int \frac{dx}{\sqrt{25-(x-4)^2}} \qquad u=x-4$$

$$\int \frac{du}{\sqrt{25-u^2}} \qquad u=5\sin\theta, \quad du=5\cos\theta\, d\theta$$

$$\int \frac{5\cos\theta\, d\theta}{\sqrt{25-25\sin^2\theta}} = \int d\theta = \theta$$
$$= \sin^{-1}\left(\frac{u}{5}\right)$$

$$\int f'(\sin x)\cos x\, dx \qquad \int f'(u)\,du = f(u)+C$$

$\int \cos^2 x\, dx$: use ½-angle identity $\cos^2 x = \frac{1}{2}(1+\cos 2x)$



① $\frac{\partial f}{\partial x} > 0$ $\quad \frac{\partial f}{\partial y} > 0$

② $\frac{\partial f}{\partial x} > 0$ $\quad \frac{\partial f}{\partial y} = 0$

$$f(x,y) = x^2 + y\sin(x)$$

$$\text{"slope in x-direction"} = \frac{\partial f}{\partial x} = f_x = 2x + y\cos(x)$$

Implicit diff: $y = y(x)$

$$x^2 + y^2 = 0$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

Improper integrals: "does $\int_0^2 \frac{1}{\sqrt{x}}\,dx$ converge and if so, what does it converge to?"

Means: compute $\lim_{t \to 0} \int_t^2 \frac{1}{\sqrt{x}}\,dx$, see if it exists or not

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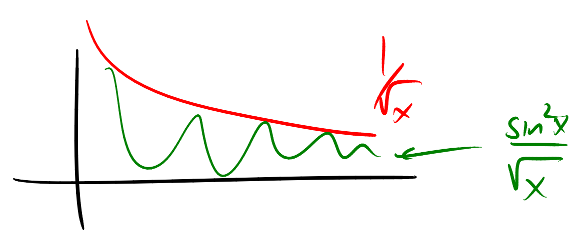
Does $\int_0^2 \frac{\sin^2 x}{\sqrt{x}}\,dx$ converge?

We know $\int_0^2 \frac{1}{\sqrt{x}}\,dx$ converges, and $0 < \frac{\sin^2 x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$



So, $\int_0^2 \frac{\sin^2 x}{\sqrt{x}}\,dx$ converges (comparison theorem)

Which $\int$'s are improper?

$$\int_a^{\infty} \qquad \int_{-\infty}^{a} \qquad \int_{-\infty}^{\infty}$$

or $\int$ of f^n w/ vertical asymptote such as $\int_1^3 \frac{1}{x-2}\,dx$

Try u-sub first:

$$\int \frac{1}{x(\ln x)^7}\,dx \qquad u = \ln x, \quad du = \frac{dx}{x}$$

$$\int x^3 \sin(x^2)\,dx \qquad u = x^2, \quad du = 2x\,dx$$

$$= \int x^2 \sin(x^2)\,(x\,dx)$$

$$= \frac{1}{2}\int u \sin(u)\,du$$

Long div:

$$\int \frac{x^3 - 4x + 1}{x^2 + 2x + 1}\,dx$$

$$\begin{array}{r|l} & x - 2 \\ \hline x^2 + 2x + 1 & x^3 + 0x^2 - 4x + 1 \\ & x^3 + 2x^2 + x \\ \hline & -2x^2 - 5x + 1 \\ & -2x^2 - 4x - 2 \\ \hline & -x + 3 \end{array}$$

$$= \int x - 2 + \frac{-x+3}{x^2+2x+1}\,dx$$