Lecture 18

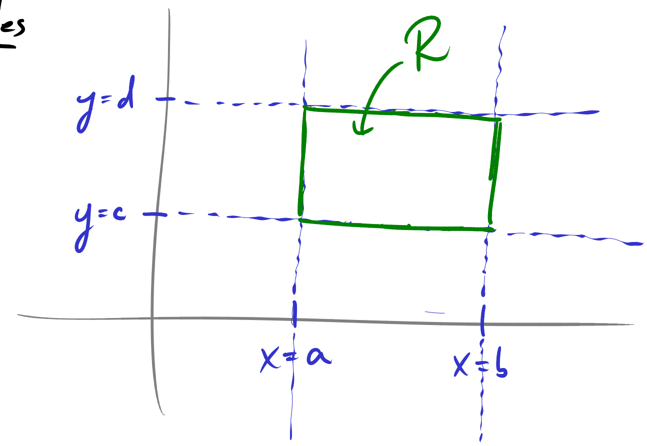
12 Oct 2012

Housekeeping: My office hour today 1pm-2pm

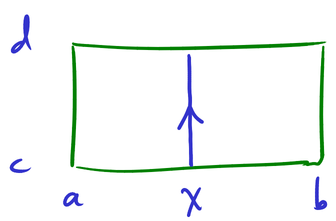
Makeup exams to be graded by end of today

Last time: double integrals over rectangles

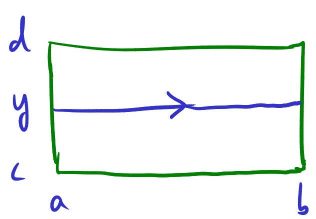
$$\iint_R f(x,y)\, dA$$



Two ways of thinking about this double integral:



$$\int_a^b \left[\int_c^d f(x,y)\, dy \right] dx$$

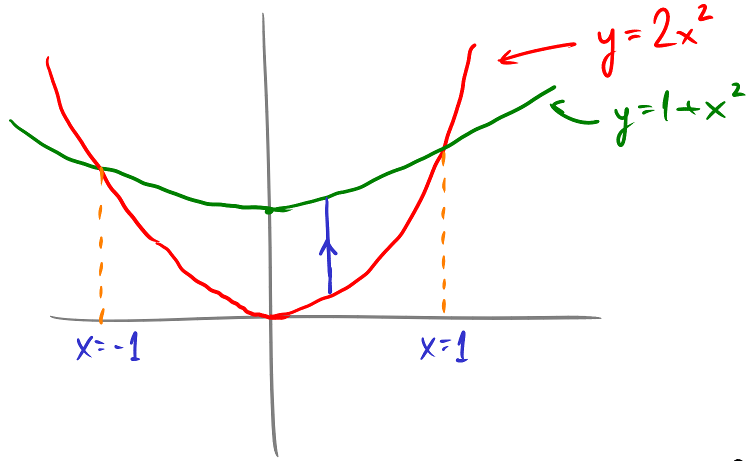


$$\int_c^d \left[\int_a^b f(x,y)\, dx \right] dy$$

Double $\int$ over Arbitrary Regions (Ch 15.3)

How about $\int$ over domains more complicated than rectangles?

Ex Evaluate $\iint_R (x+2y)\, dA$ where R is the domain between the curves $y=2x^2$ and $y=1+x^2$.



$2x^2=1+x^2 \Rightarrow x=\pm 1$

Our double integral is

$$\int_{-1}^{1}\left[\int_{2x^2}^{1+x^2}(x+2y)\,dy\right]dx$$

$$=\int_{-1}^{1}\left[xy+y^2\right]_{y=2x^2}^{y=1+x^2}\,dx$$

$$=\int_{-1}^{1}\left(x(1+x^2)+(1+x^2)^2\right)-\left(x(2x^2)+(2x^2)^2\right)\,dx$$

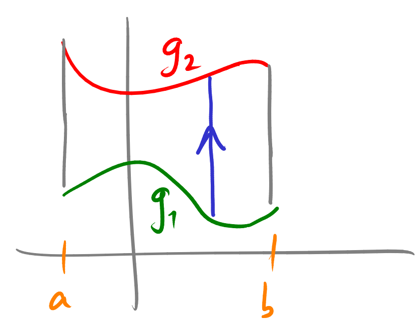
$$=\int_{-1}^{1}x+x^3+1+2x^2+x^4-2x^3-4x^4\,dx$$

$$=\int_{-1}^{1}1+x+2x^2-x^3-3x^4\,dx=\cdots=\frac{32}{15}$$

We use this method whenever we have a domain of the form

$$R=\{(x,y):\ a\le x\le b,\ \ g_1(x)\le y\le g_2(x)\}$$

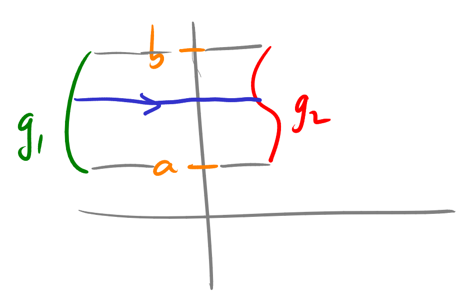
$$\iint_R f(x,y)\,dx\,dy=\int_a^b\left[\int_{g_1(x)}^{g_2(x)}f(x,y)\,dy\right]dx$$



If

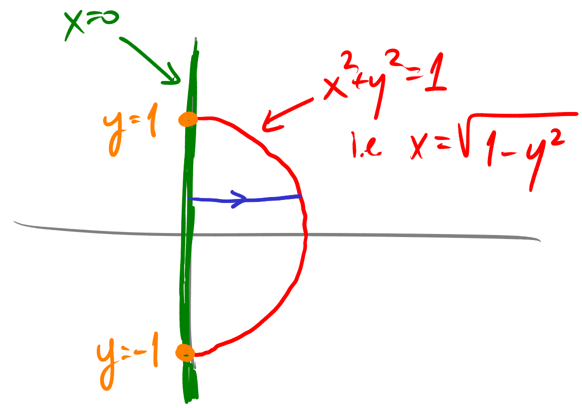
$$R=\{(x,y):\ a\le y\le b\ \ g_1(y)\le x\le g_2(y)\}$$

$$\iint_R f(x,y)\,dx\,dy=\int_a^b\left[\int_{g_1(y)}^{g_2(y)}f(x,y)\,dx\right]dy$$



Ex $\iint_R xy^2 \, dA$ where R = the domain enclosed by the line $x=0$, the circle $x^2+y^2=1$, with $x>0$

$$\int_{-1}^{1}\left[\int_{0}^{\sqrt{1-y^2}} xy^2 \, dx\right] dy$$

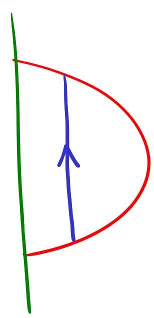


$$= \int_{-1}^{1}\left(\frac{1}{2}x^2y^2 \Big]_{x=0}^{x=\sqrt{1-y^2}}\right) dy$$

$$= \int_{-1}^{1} \frac{1}{2}y^2\left((1-y^2)-0^2\right) dy$$

$$= \int_{-1}^{1} \frac{1}{2}y^2 - \frac{1}{2}y^4 \, dy = \cdots = \underline{\underline{2/15}}$$

[Alternative:



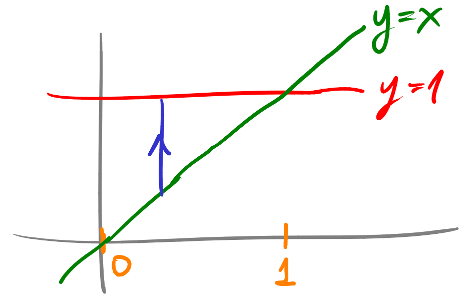
$$\int_0^1\left(\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} xy^2 \, dy\right) dx$$
]

Ex $\int_0^1 \left[\int_x^1 \sin(y^2) \, dy\right] dx$

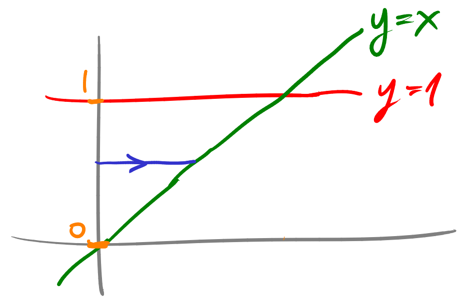
This looks hard – <u>can't do</u> the $\int$ over y!

Interpret it as a double $\int$:

$$\iint_D \sin(y^2)\; dA$$



Try $\int$ over x first:



$$\int_0^1 \left[\int_0^y \sin(y^2)\; dx \right] dy$$

$$= \int_0^1 \left(x \sin(y^2) \Big]_{x=0}^{x=y} \right) dy$$

$$= \int_0^1 y \sin(y^2)\; dy$$

$u = y^2$
$du = 2y\, dy$

$$= \int_0^1 \sin(u) \cdot \tfrac{1}{2}\, du$$

$$= -\tfrac{1}{2} \cos(u) \Big|_0^1 = -\tfrac{1}{2}\left(\cos(1) - 1\right)$$