

$$\underline{\text{Ex}} \quad I = \int \sec^3 x \, dx$$

$$\text{Int. by parts:} \quad u = \sec x \quad v = \tan x \\ du = \sec x \tan x \, dx \quad dv = \sec^2 x \, dx$$

$$I = \int u \, dv = uv - \int v \, du \\ = \sec x \tan x - \int \tan x \sec x \tan x \, dx \\ = \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$\text{Use } \tan^2 x = \sec^2 x - 1$$

$$I = \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$$

$$I = \sec x \tan x + \int \sec x \, dx - \int \sec^3 x \, dx$$

$$I = \sec x \tan x + \ln |\sec x + \tan x| + C - I$$

$$2I = \sec x \tan x + \ln |\sec x + \tan x| + C$$

$$I = \frac{1}{2} (\sec x \tan x + \ln |\sec x + \tan x|) + C$$
