March 31^{st} 2010

SEQUENCES

<u>Def</u>: Sequence: list of numbers in order:

$$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad \cdots\cdots \quad a_n \cdots\cdots$$

$$n: \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \cdots \qquad 100,\ 101,\ \cdots$$

Given a general sequence $\{a_n\}_{n \in \mathbb{N}}$,

$\boxed{\text{" } \in \text{ " belongs}}$

<u>ex</u>: take $a_n = 2n$ sequence of even numbers

$$0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \quad \cdots$$

$$a_{25} = 50$$

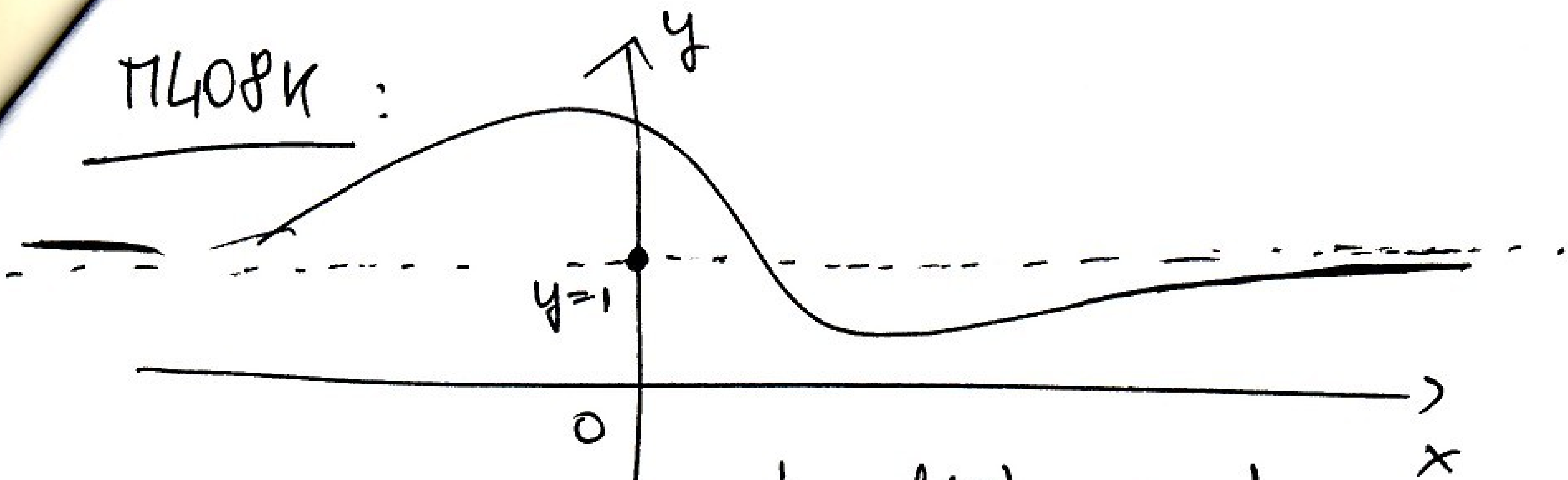
$$a_{25} = 2(25) = 50$$

$$a_n = n^2$$

$$0 \quad 1 \quad 4 \quad 9 \quad 16 \quad 25 \quad 36$$

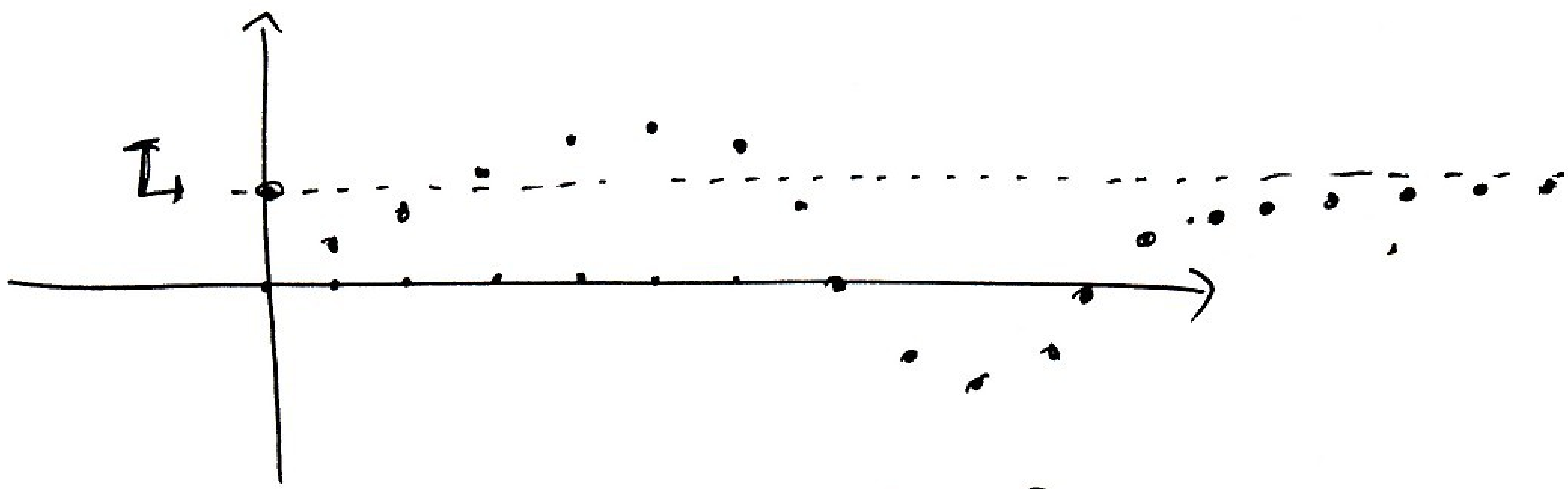
$$a_{10} = (10)^2 = 100$$

M408K :

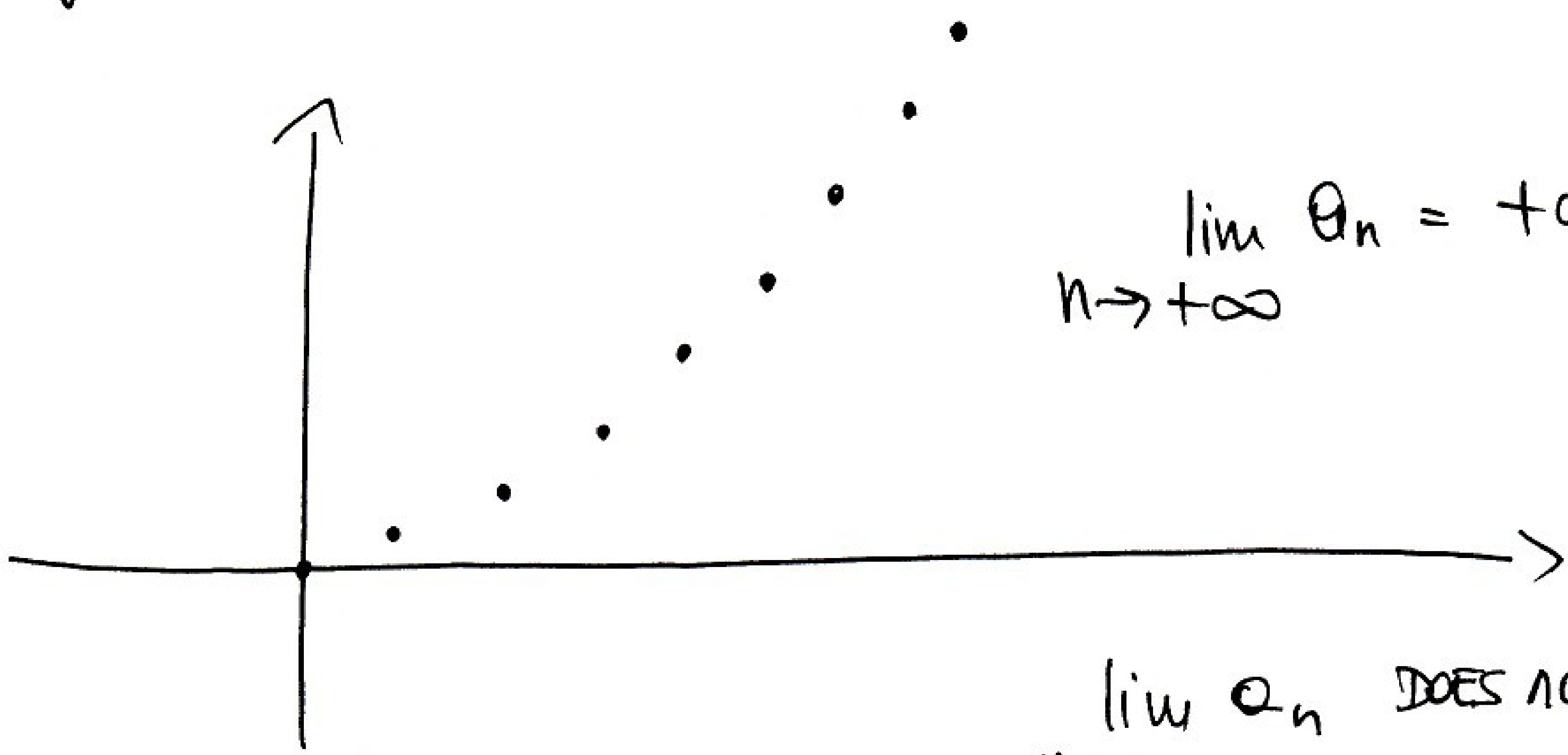


$$\lim_{x \to \pm\infty} f(x) = 1$$

$y = 1$ horizontal asymptote

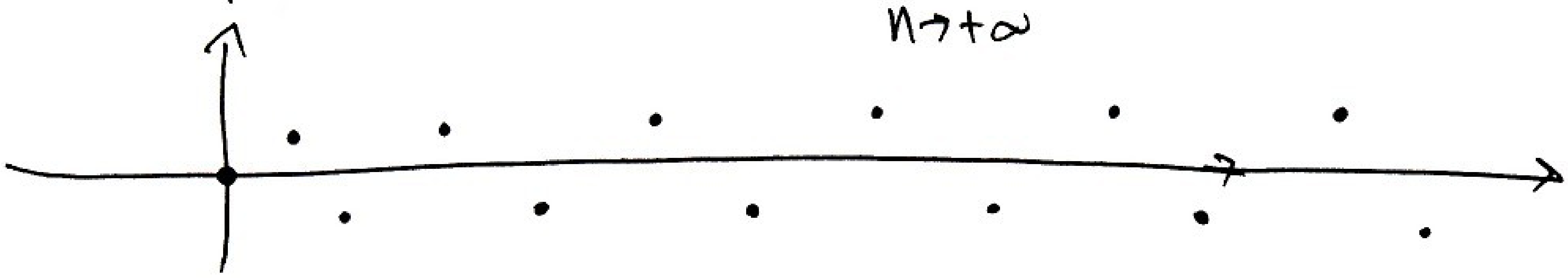


$y = L$ = limit of $\{a_n\}$ as $n \to +\infty$



$$\lim_{n \to +\infty} a_n = +\infty$$

$\lim_{n \to +\infty} a_n$ DOES NOT EXIST

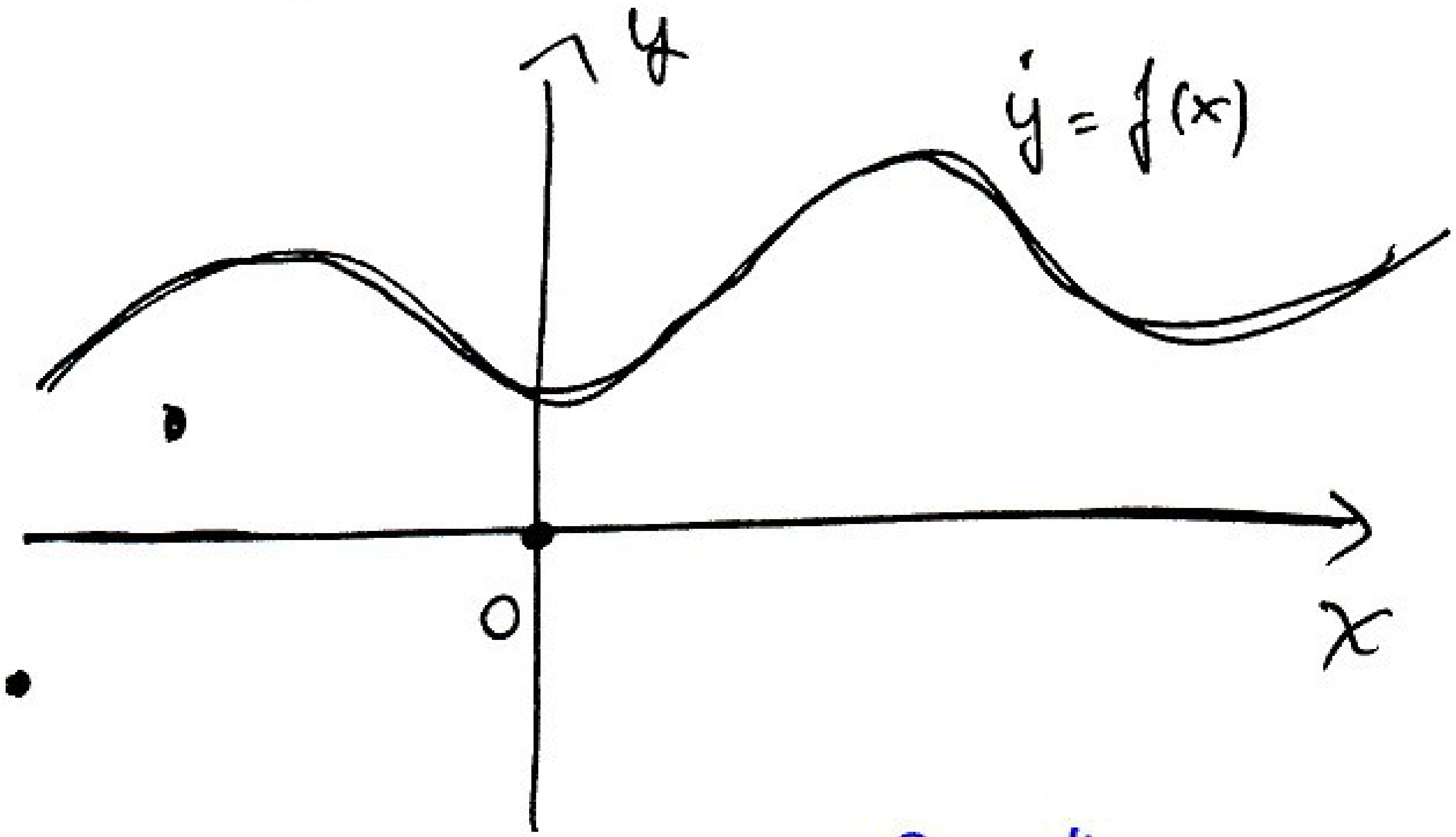


$$a_n = \frac{n}{n+1}$$

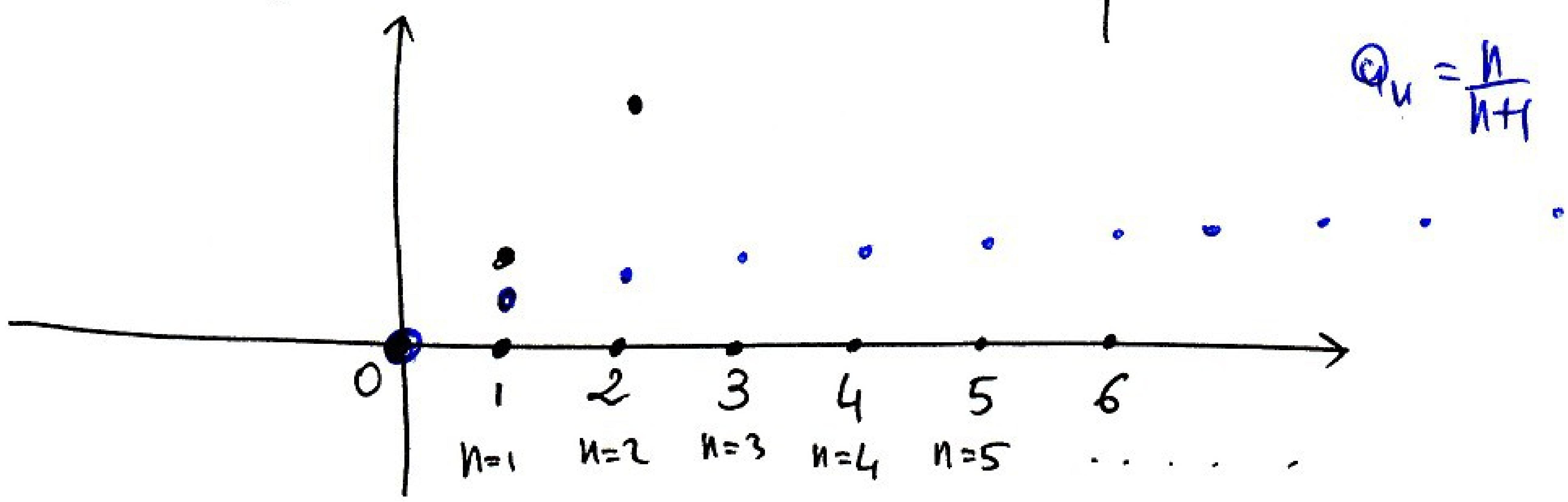
$0 \quad \frac{1}{2} \quad 2/3 \quad 3/4 \quad 4/5 \quad 5/6 \quad 6/7 \quad \ldots$

$$a_{17} = 17/18$$

Sequences $\Longleftrightarrow$ functions



Can we graph a sequence?



<u>DeF</u>: A sequence $\{a_n\}_{n\in\mathbb{N}}$ converges to a limit L if

$$\lim_{n\to+\infty} a_n = L$$

$\{a_n\}$ is CONVERGENT to L

"a_n approaches L as $n \nearrow +\infty$
a_n is close to L as much as you want for $n \to +\infty$".

<u>DeF</u>: If such <u>limit L</u> does <u>not exist</u>
$\Rightarrow$ $\{a_n\}_{n\in\mathbb{N}}$ is **DIVERGENT**

ex: $a_n = \dfrac{1}{n}$

$1 \quad \frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6} \quad \cdots\cdots$

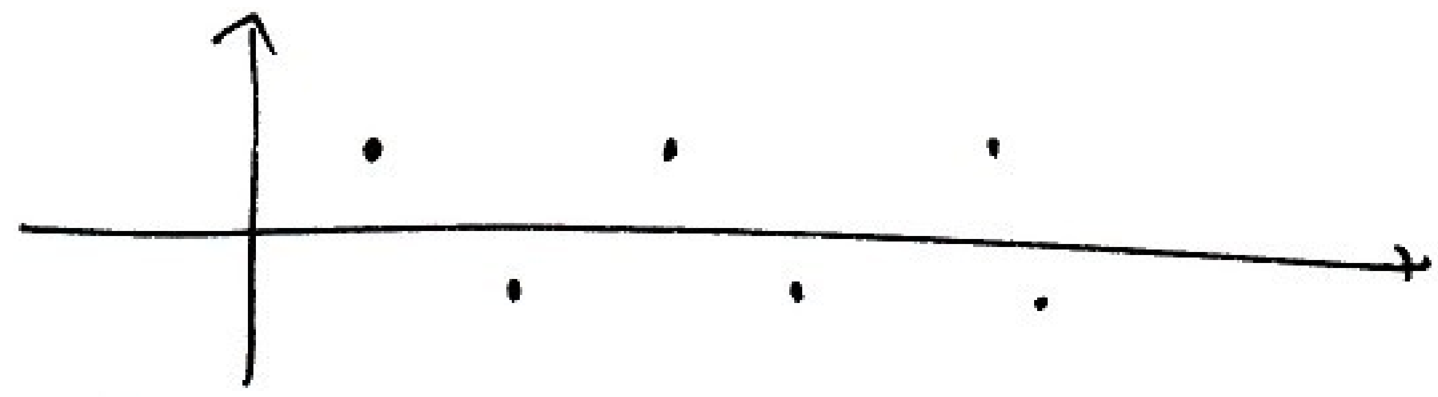
$$\lim_{n\to+\infty} \frac{1}{n} = 0$$

ex: $a_n = \dfrac{n}{3n+1}$

$$\lim_{n\to+\infty} \frac{\frac{n}{n}}{\frac{3n}{n}+\frac{1}{n}} = \lim_{n\to+\infty} \frac{1}{3+\frac{1}{n}} = \frac{1}{3}$$

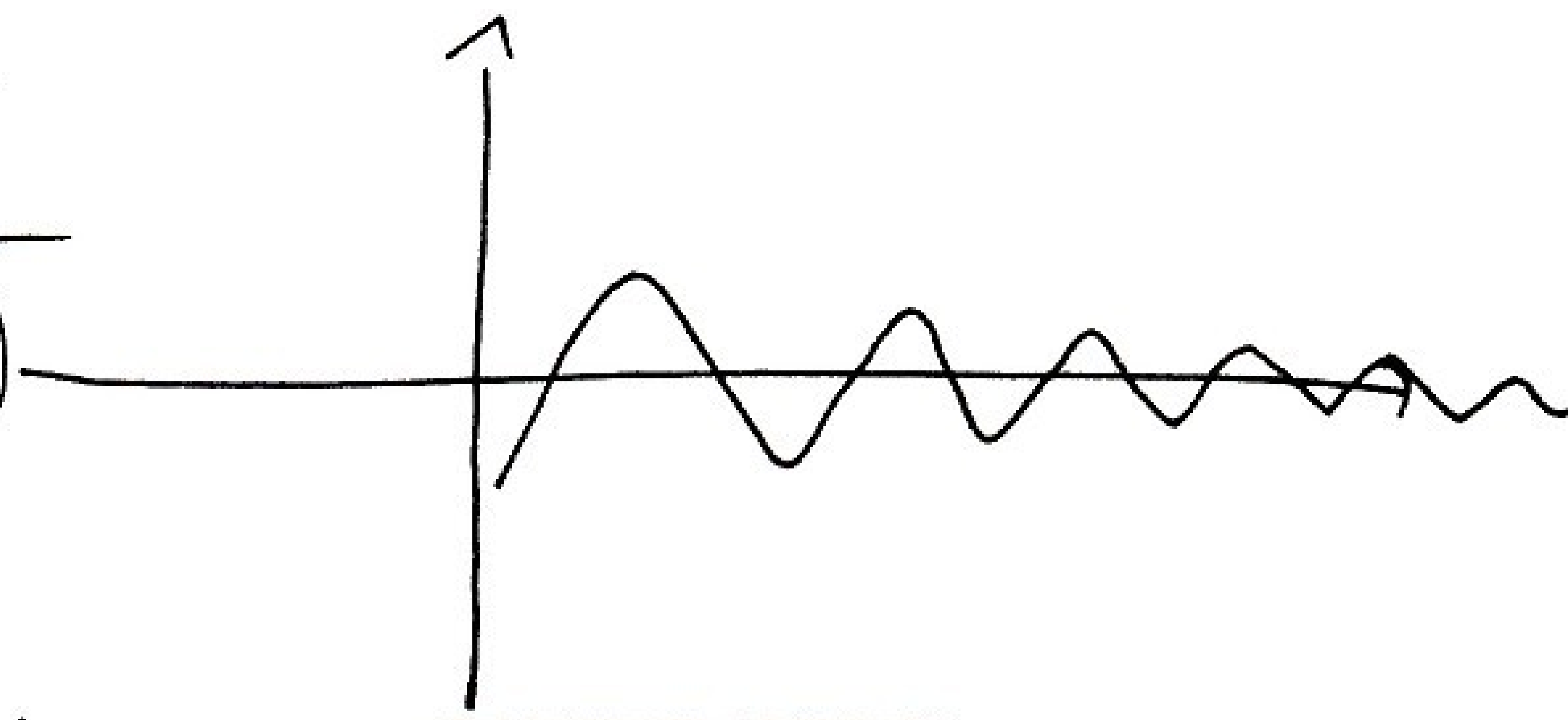
$\Rightarrow \left\{\dfrac{n}{3n+1}\right\}_{n\in\mathbb{N}}$ is convergent.

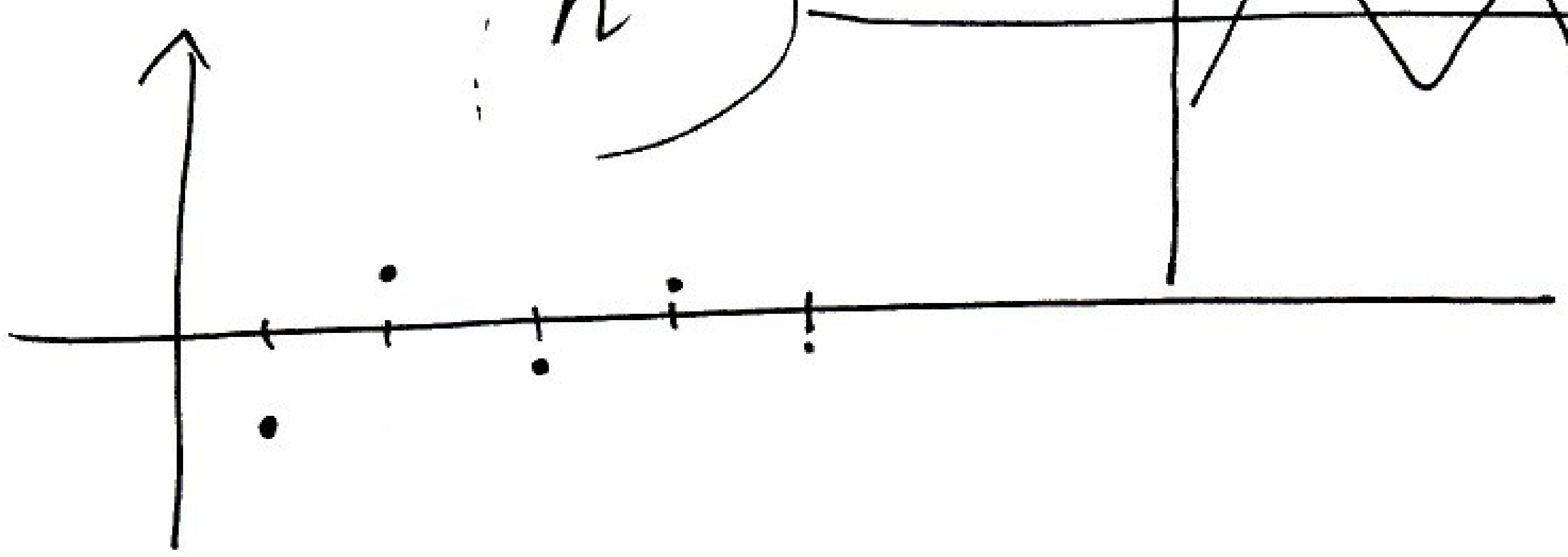
ex: $a_n = (-1)^n$



$\lim\limits_{n\to+\infty} (-1)^n$ does not exist!

ex: $a_n = \dfrac{(-1)^n}{n^2}$





$$\lim_{n\to+\infty} \frac{(-1)^n}{n^2} = 0$$

ex: $a_n = \frac{\ln(n)}{n}$?

Back to functions: $\frac{\ln(x)}{x}$

$$\lim_{x\to+\infty} \frac{\ln(x)}{x} = $$

$$= \lim_{x\to+\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to+\infty} \frac{1}{x} = 0$$

$\Downarrow$

$$\lim_{n\to+\infty} \frac{\ln(n)}{n} = 0$$

ex:

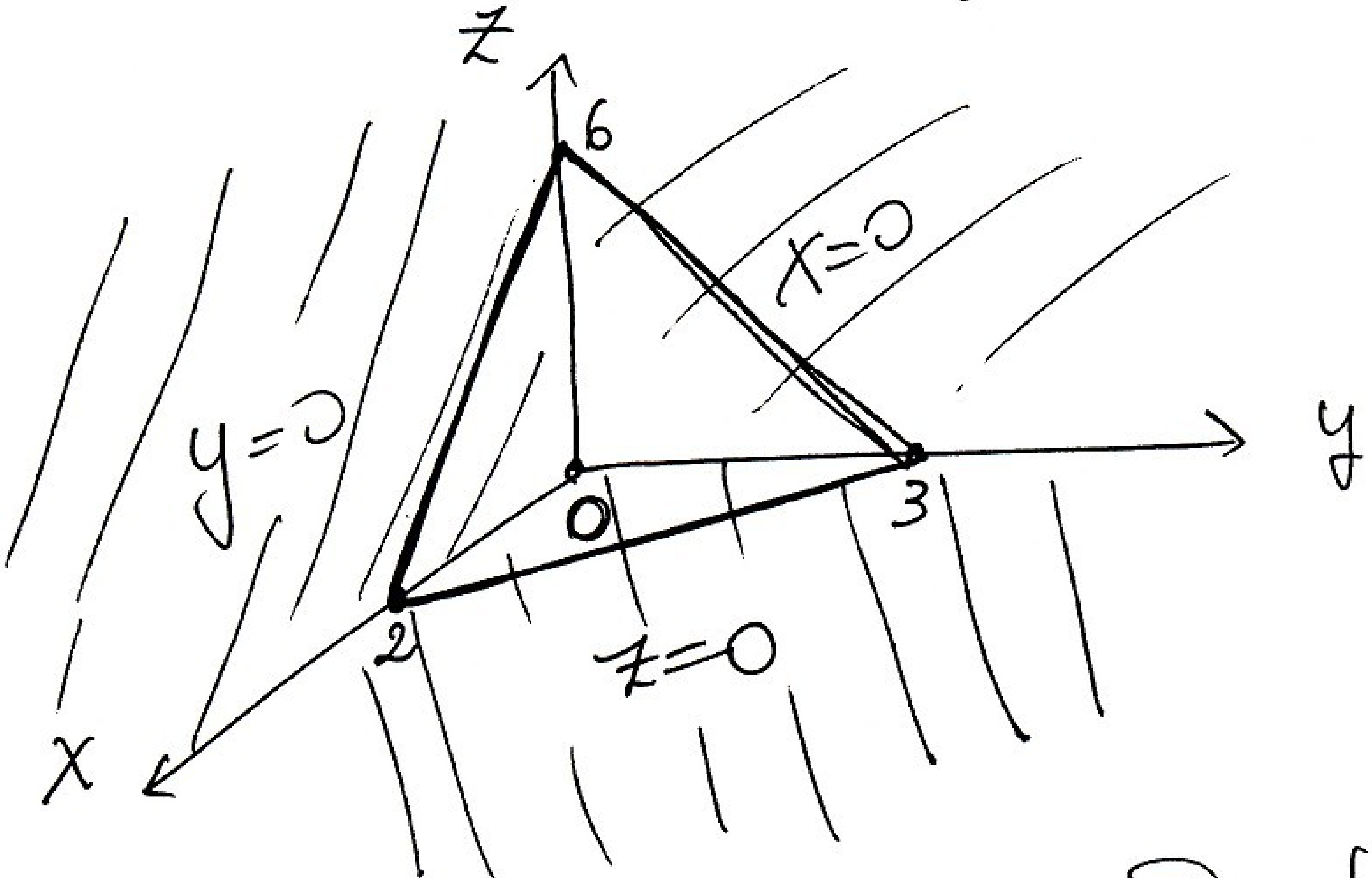
$$a_n = \frac{1}{n^2} \longrightarrow 0$$

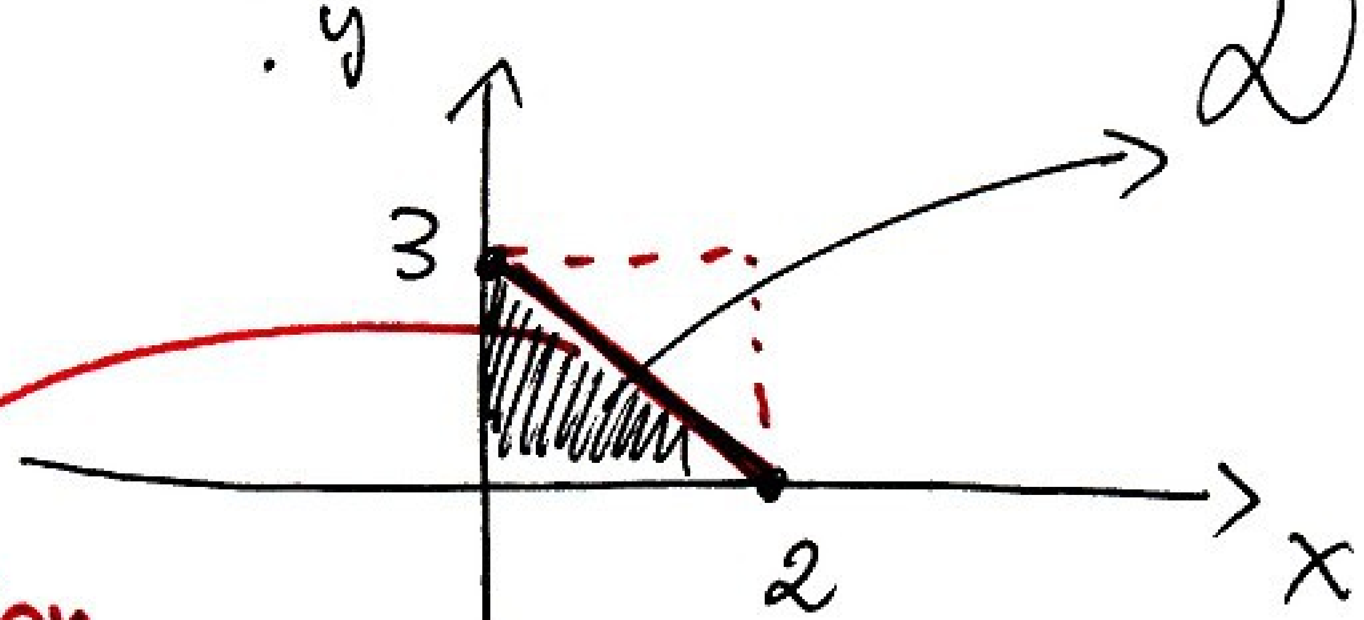
Double Integrals

ex: Volume of the solid bounded by:

(1) coordinate planes $\begin{cases} z = 0 \\ y = 0 \\ x = 0 \end{cases}$

(2) plane $3x + 2y + z = 6$





$\mathcal{D} = \{ (x, y)$ such that
$0 \le x \le 2$
$0 \le y \le -\frac{3}{2}x +$

intersection of plane $3x + 2y + z = 6$ with "floor" $\equiv (z = 0)$

$\Downarrow$

$3x + 2y = 6 \Rightarrow y = -\frac{3}{2}x + 3$

$$\iint_{D} \text{"plane"}\, dx\,dy = \iint_{D} (6-3x-2y)\, dx\,dy$$

$$3x+2y+z=6$$

$$\Downarrow$$

$$z = 6-3x-2y$$

$$D = \{(x,y) : \quad 0 \le x \le 2$$
$$0 \le y \le -\tfrac{3}{2}x+3$$

$$\int_0^2 \left(\int_0^{-\frac{3}{2}x+3} (6-3x-2y) \quad dy \right) dx$$

$\Rightarrow$ Solve the double integral:

$$\int_0^2 \left(\int_0^{-\frac{3}{2}x+3} (6-3x-2y)\, dy \right) dx =$$

$$= \int_0^2 \left(6y - 3xy - y^2 \Big|_0^{-\frac{3}{2}x+3} \right) dx$$

$$= \int_0^2 \left(6\left(-\tfrac{3}{2}x+3\right) - 3x\left(-\tfrac{3}{2}x+3\right) - \left(-\tfrac{3}{2}x+3\right)^2 \right) dx$$

$$= \int_0 \left(-9x + 18 + \frac{9}{2}x^2 - 3x - \frac{9}{4}x^2 - 9 + 9x\right) dx$$

$= \ldots\ldots$ $= \underline{\text{NUMBER}}$