

Practice Midterm 1 for Math 341

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- Let $\vec{v} = [1, 2]$ and $\vec{w} = [2, 3]$.
 - Calculate $\vec{v} + \vec{w}$, and draw the diagram corresponding to the vector addition in the xy -plane.
 - Calculate $\cos(\theta)$, where θ is the angle between \vec{v} and \vec{w} .
 - Check that \vec{v} and \vec{w} satisfy the triangle inequality.
 - Calculate the unit vector in the direction of $\vec{v} - 2\vec{w}$.
 - Calculate $\text{proj}_{\vec{v}}\vec{w}$.
- Let \vec{x} and \vec{y} be a pair of vectors in \mathbb{R}^n .
 - Prove that $\vec{x} + \vec{y} = \vec{y} + \vec{x}$.
 - Show that if $\vec{x} - 2\vec{y}$ and $\vec{x} + 2\vec{y}$ are orthogonal, then $\|\vec{x}\| = 2\|\vec{y}\|$.
- Let the matrices A, B and C be defined as:

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}, C = \begin{bmatrix} -1 & 1 & 2 \\ 2 & 4 & 10 \\ 1 & 0 & 2 \end{bmatrix},$$

Calculate the matrix expressions below, if they are possible; if they are not possible, explain why not.

- AB
 - BA
 - $BC + 2C$
 - $A + B$
- Let A be a $n \times 3$ matrix with columns \vec{v}_1, \vec{v}_2 and \vec{v}_3 in that order. For which vector \vec{x} is $A\vec{x} = 2\vec{v}_1 + \vec{v}_2 - 3\vec{v}_3$?
 - Put the following systems of equations into row-reduced echelon form, and use this to find all solutions. State how many solutions each system has.

(a)

$$a + b + 3c = 1$$

$$a + 2b - c = 3$$

(b)

$$3a + 2b = 1$$

$$2a + b = 1$$

6. Does there exist a system of linear equations with exactly 3 solutions? If yes, provide an example; if not, explain why not.
7. Let A and B be two matrices.
- (a) If A is both upper triangular and lower triangular, show that A must be diagonal.
 - (b) If $A^2 = B$, show that A and B must both be square matrices.
 - (c) Show that if A is upper triangular and B is diagonal, then AB is upper triangular.