Questions for First Proof Class

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The following are the questions for the first proof class. Before the class, do your best to write up a proof for each of these questions. You may be called on to present them!

1. Show that if A and B are two $n \times n$ diagonal matrices, then $AB = BA$.
2. Prove that if A is an $m \times n$ matrix with columns $\vec{v}_1, \ldots, \vec{v}_n$, and

$$\vec{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

   then

$$A\vec{c} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n$$

3. Let A be an $m \times n$ matrix with rows $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_m$.
   (a) Show that if $\vec{x}$ is orthogonal to $\vec{r}_i$ for each i (that is, orthogonal to every single row of A), then $A\vec{x} = \vec{0}$.
   (b) Show that if $\vec{x}$ and $\vec{y}$ are in the row space of A, then so is $\vec{x} + \vec{y}$.