Midterm 1: Concepts to Review

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The first midterm will cover Sections 1.5, 1.6, 2.2, 2.3, 2.5, 2.6, as well as Appendix D – basically everything we did in lecture up until Thursday, September 15th. In addition, any precalculus concepts that we've been using in lecture and on the homework will be tested implicitly!

1. Trigonometry (Appendix D)
   - Using the unit circle to find $\sin(\theta)$ and $\cos(\theta)$
   - Definitions of the trig functions, such as $\sin, \cos, \tan, \cot, \csc, \sec$
   - Formulas such as
   $$\begin{aligned} \sin^2(\theta) + \cos^2(\theta) &= 1 \\ \tan^2(\theta) + 1 &= \sec^2(\theta) \\ 1 + \cot^2(\theta) &= \csc^2(\theta) \end{aligned}$$
   - Angle addition formulas like $\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$, and versions for $\cos, \tan$, etc.
   - Values of the trig functions at the angles in the table on page A27: that is, multiples of $\frac{\pi}{4}$ and multiples of $\frac{\pi}{6}$
2. Exponentials (Section 1.5)
   - The definition of $f(x) = a^x$.
   - Laws of exponents (and using them to solve problems)
   $$a^{x+y} = a^x a^y,\ a^{x-y} = \frac{a^x}{a^y},\ a^{xy} = (a^x)^y,\ (ab)^x = a^x b^x$$
   - Graphs of exponential functions
3. Inverse Functions and Logs (Section 1.6)
   - One-to-one functions
   - The definition of $f^{-1}(x)$ if $f(x)$ is one-to-one, domain and range of f^{-1}, graphing f^{-1} given the graph of f
   - Cancellation equations:
   $$f(f^{-1}(x)) = x,\ f^{-1}(f(x)) = x$$

- Finding an explicit formula for the inverse of a function
- Logarithms, logarithm laws, the number e and the natural log (ln):

$$\log_a(xy) = \log_a(x) + \log_a(y), \quad \log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y),$$
$$\log_a(x^r) = r\log_a(x), \quad \log_a(x) = \frac{\ln x}{\ln a}$$

- Inverse trigonometric functions: definitions and graphs

4. The limit of a function (Section 2.2)

- The concept of a limit: what it means when

$$\lim_{x\to a} f(x) = L$$

- Using the graph of a function to determine a limit
- One-sided limits (knowing what the following mean):

$$\lim_{x\to a^+} f(x) = L, \lim_{x\to a^-} f(x) = L$$

- If $f(x) \le g(x)$ near (except possibly at) a, and both the limits exist, then

$$\lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$$

- The Squeeze Theorem: if $f(x) \le g(x) \le h(x)$ near (except possibly at) a, and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then

$$\lim_{x\to a} g(x) = L$$

- Infinite limits: the meanings of

$$\lim_{x\to a} f(x) = \infty, \lim_{x\to a} f(x) = -\infty$$

and variants with one-sided limits, such as $\lim_{x\to a^+} f(x) = \infty$.
- Vertical asymptotes of functions

5. Calculating limits using limit laws (Section 2.3)

- If c is a constant, and $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then:

$$\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$
$$\lim_{x\to a}(f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$
$$\lim_{x\to a}(cf(x)) = c\lim_{x\to a} f(x)$$
$$\lim_{x\to a}(f(x)g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$
$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \text{ if } \lim_{x\to a} g(x) \ne 0$$

- Other laws that follow from above in Section 2.3 (see textbook for full list)
- Being able to use the above laws to calculate limits
- Knowing when *not* to use these limit laws: if $\lim_{x \to a} f(x)$ or $\lim_{x \to a} g(x)$ don't exist, we can't use these. Remember that just because these don't exist does NOT mean that, say, $\lim_{x \to a}(f(x) + g(x))$ doesn't exist– you just can't use the laws to calculate it!

6. Continuity (Section 2.5)

- The definition of being continuous at a point: $f(x)$ is continuous at a if
$$\lim_{x \to a} f(x) = f(a)$$
- Similar definitions for $f(x)$ being continuous at a from the left and from the right.
- Using the graph of a function to determine where it's continuous
- If f and g are continuous at a, then the following functions are also continuous at a:
$$f + g, \ f - g, \ fg, \ \frac{f}{g} \text{ if } g(a) \neq 0$$
- Knowing functions that are continuous everywhere on their domain, and using this fact to calculate limits. These include: polynomials, rational functions, root functions, exponentials, logarithms, trig functions, inverse trig functions.
- If f is continuous at b, and $\lim_{x \to a} g(x) = b$, then
$$\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x)) = f(b)$$
- Using the Intermediate Value Theorem to show that equations have a solution (without finding the solution!)

7. Limit at infinity (Section 2.6)

- The meanings of
$$\lim_{x \to \infty} f(x) = L, \ \lim_{x \to -\infty} f(x) = L$$
- Horizontal asymptotes of functions
- Rules such as
$$\lim_{x \to \infty} \frac{1}{x^r} = 0$$
for rational numbers $r > 0$.
- Manipulating expressions in order to calculate limits at infinity