

MATH 408N PRACTICE MIDTERM 2

Name: _____

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TA session: _____

Show your work for all the problems. Good luck!

(1) Use the limit definition of the derivative for the following questions. *You will get no points for using the rules learned later!*

(a) [5 pts] Find $f'(1)$ if $f(x) = \sqrt{x}$.

(b) [5 pts] Find $f'(x)$ if $f(x) = x^2 + x + 1$.

(2) Differentiate the following functions, using whatever methods you think are best (you are now allowed to use all the rules):

(a) [5 pts] $f(x) = x^3 + 3x + 5$

(b) [5 pts] $f(x) = \arctan(x^2) e^x$

(c) [5 pts] $f(x) = \frac{2x \sin(x)}{\ln(x+1)}$

(d) [5 pts] $f(x) = \sin(x)^{\cos(x)}$

(3) Find the equations of the tangent lines to the following graphs at the given points:

(a) [7 pts] $y = x^2 \ln(x) + 1$ at $(1, 1)$.

(b) [8 pts] $x^2 + y^2 = xe^y$ at $(1, 0)$.

(4) Let $f(x)$ and $g(x)$ satisfy $f(1) = 3$, $g(1) = 2$, $f'(1) = 1$, $g'(1) = -1$, and $f'(2) = -2$.

(a) [5 pts] If $F(x) = f(x)g(x)$, calculate $F'(1)$.

(b) [5 pts] If $G(x) = f(g(x))$, calculate $G'(1)$.

(c) [5 pts] If $H(x) = g(f(x))$, do we have enough information to calculate $H'(1)$?

(5) For the following graphs, find all the points at which the tangent line to the graph is parallel to $y = 2x + 3$. *Find both coordinates of each point for full marks!*

(a) [5 pts] $y = x^2 + 2x + 3$.

(b) [5 pts] $x^2 + y^2 = 5$.

(6) Find y'' for the following graphs:

(a) [7 pts] $y = \sin(\cos(x))$

(b) [8 pts] $x^2 + xy + y^2 = 2$ (Here, your answer can include both x and y .)

(7) A 25 foot ladder is sliding down a vertical wall. The top of the ladder is sliding down the wall at the rate of 1 ft/sec. (*You should express the answers to these questions as fractions: no need to make them decimals!*)

(a) [7 pts] How quickly is the bottom of the ladder moving away from the vertical wall when the top of the ladder is 20 feet away from the floor?

(b) [8 pts] How quickly is the angle between the ladder and the floor (i.e. the horizontal) changing when the top of the ladder is 20 feet away from the floor? Is it increasing or decreasing?

(8) Answer the following questions:

(a) [5 pts] Find the linearization $L(x)$ to $f(x) = \frac{1}{\sqrt{x+1}}$ at $x = 3$.

(b) [5 pts] Use linearization to provide an estimate for $\sqrt[3]{29}$, given that $\sqrt[3]{27} = 3$.