Homework 11

Section 3.4:

8. $F(x) = (4x - x^2)^{100} \quad \Rightarrow \quad F'(x) = 100(4x - x^2)^{99} \cdot \dfrac{d}{dx}(4x - x^2) = 100(4x - x^2)^{99}(4 - 2x)$

$\left[\text{or } 200x^{99}(x-2)(x-4)^{99}\right]$

18. $g(x) = (x^2+1)^3(x^2+2)^6 \quad \Rightarrow$

$$g'(x) = (x^2+1)^3 \cdot 6(x^2+2)^5 \cdot 2x + (x^2+2)^6 \cdot 3(x^2+1)^2 \cdot 2x$$
$$= 6x(x^2+1)^2(x^2+2)^5[2(x^2+1) + (x^2+2)] = 6x(x^2+1)^2(x^2+2)^5(3x^2+4)$$

24. Using Formula 5 and the Chain Rule, $y = 10^{1-x^2} \quad \Rightarrow \quad y' = 10^{1-x^2}(\ln 10) \cdot \dfrac{d}{dx}\left(1 - x^2\right) = -2x(\ln 10)10^{1-x^2}$.

28. $y = \dfrac{e^u - e^{-u}}{e^u + e^{-u}} \quad \Rightarrow$

$$y' = \frac{(e^u + e^{-u})(e^u - (-e^{-u})) - (e^u - e^{-u})(e^u + (-e^{-u}))}{(e^u + e^{-u})^2} = \frac{e^{2u} + e^0 + e^0 + e^{-2u} - (e^{2u} - e^0 - e^0 + e^{-2u})}{(e^u + e^{-u})^2}$$
$$= \frac{4e^0}{(e^u + e^{-u})^2} = \frac{4}{(e^u + e^{-u})^2}$$

42. $y = \sqrt{x + \sqrt{x + \sqrt{x}}} \quad \Rightarrow \quad y' = \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{-1/2}\left[1 + \frac{1}{2}\left(x + \sqrt{x}\right)^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)\right]$

44. $y = 2^{3^{x^2}} \quad \Rightarrow \quad y' = 2^{3^{x^2}}(\ln 2)\dfrac{d}{dx}\left(3^{x^2}\right) = 2^{3^{x^2}}(\ln 2)3^{x^2}(\ln 3)(2x)$

52. $y = \sqrt{1 + x^3} = (1 + x^3)^{1/2} \quad \Rightarrow \quad y' = \frac{1}{2}(1 + x^3)^{-1/2} \cdot 3x^2 = \dfrac{3x^2}{2\sqrt{1 + x^3}}$. At $(2, 3)$, $y' = \dfrac{3 \cdot 4}{2\sqrt{9}} = 2$, and an equation of the tangent line is $y - 3 = 2(x - 2)$, or $y = 2x - 1$.

72. $f(x) = xg(x^2) \quad \Rightarrow \quad f'(x) = xg'(x^2)\,2x + g(x^2) \cdot 1 = 2x^2g'(x^2) + g(x^2) \quad \Rightarrow$

$$f''(x) = 2x^2g''(x^2)\,2x + g'(x^2)\,4x + g'(x^2)\,2x = 4x^3g''(x^2) + 4xg'(x^2) + 2xg'(x^2) = 6xg'(x^2) + 4x^3g''(x^2)$$