Homework 13

Section 3.5:

22. $\dfrac{d}{dx}\left[g(x)+x\sin g(x)\right]=\dfrac{d}{dx}\left(x^2\right) \;\Rightarrow\; g'(x)+x\cos g(x)\cdot g'(x)+\sin g(x)\cdot 1=2x.$ If $x=0$, we have

$g'(0)+0+\sin g(0)=2(0) \;\Rightarrow\; g'(0)+\sin 0=0 \;\Rightarrow\; g'(0)+0=0 \;\Rightarrow\; g'(0)=0.$

26. $\sin(x+y)=2x-2y \;\Rightarrow\; \cos(x+y)\cdot(1+y')=2-2y' \;\Rightarrow\; \cos(x+y)\cdot y'+2y'=2-\cos(x+y) \;\Rightarrow$

$y'[\cos(x+y)+2]=2-\cos(x+y) \;\Rightarrow\; y'=\dfrac{2-\cos(x+y)}{\cos(x+y)+2}$. When $x=\pi$ and $y=\pi$, we have $y'=\dfrac{2-1}{1+2}=\dfrac{1}{3}$, so an equation of the tangent line is $y-\pi=\frac{1}{3}(x-\pi)$, or $y=\frac{1}{3}x+\frac{2\pi}{3}$.

52. $g(x)=\sqrt{x^2-1}\,\sec^{-1}x \;\Rightarrow\; g'(x)=\sqrt{x^2-1}\cdot\dfrac{1}{x\sqrt{x^2-1}}+\sec^{-1}x\cdot\dfrac{1}{2}\left(x^2-1\right)^{-1/2}(2x)=\dfrac{1}{x}+\dfrac{x\sec^{-1}x}{\sqrt{x^2-1}}$

$\left[\text{or } \dfrac{\sqrt{x^2-1}+x^2\sec^{-1}x}{x\sqrt{x^2-1}}\right]$

56. $F(\theta)=\arcsin\sqrt{\sin\theta}=\arcsin(\sin\theta)^{1/2} \;\Rightarrow$

$F'(\theta)=\dfrac{1}{\sqrt{1-\left(\sqrt{\sin\theta}\right)^2}}\cdot\dfrac{d}{d\theta}(\sin\theta)^{1/2}=\dfrac{1}{\sqrt{1-\sin\theta}}\cdot\dfrac{1}{2}(\sin\theta)^{-1/2}\cdot\cos\theta=\dfrac{\cos\theta}{2\sqrt{1-\sin\theta}\sqrt{\sin\theta}}$

Section 3.6:

32. $f(x) = \ln(1+e^{2x}) \;\Rightarrow\; f'(x) = \dfrac{1}{1+e^{2x}}(2e^{2x}) = \dfrac{2e^{2x}}{1+e^{2x}}$, so $f'(0) = \dfrac{2e^0}{1+e^0} = \dfrac{2(1)}{1+1} = 1.$

40. $y = \dfrac{e^{-x}\cos^2 x}{x^2+x+1} \;\Rightarrow\; \ln y = \ln \dfrac{e^{-x}\cos^2 x}{x^2+x+1} \;\Rightarrow$

$\ln y = \ln e^{-x} + \ln |\cos x|^2 - \ln(x^2+x+1) = -x + 2\ln|\cos x| - \ln(x^2+x+1) \;\Rightarrow$

$\dfrac{1}{y}y' = -1 + 2\cdot\dfrac{1}{\cos x}(-\sin x) - \dfrac{1}{x^2+x+1}(2x+1) \;\Rightarrow\; y' = y\left(-1 - 2\tan x - \dfrac{2x+1}{x^2+x+1}\right) \;\Rightarrow$

$y' = -\dfrac{e^{-x}\cos^2 x}{x^2+x+1}\left(1 + 2\tan x + \dfrac{2x+1}{x^2+x+1}\right)$

46. $y = \sqrt{x}^{\,x} \;\Rightarrow\; \ln y = \ln \sqrt{x}^{\,x} \;\Rightarrow\; \ln y = x\ln x^{1/2} \;\Rightarrow\; \ln y = \frac{1}{2}x\ln x \;\Rightarrow\; \dfrac{1}{y}y' = \dfrac{1}{2}x\cdot\dfrac{1}{x} + \ln x\cdot\dfrac{1}{2} \;\Rightarrow$

$y' = y\left(\frac{1}{2} + \frac{1}{2}\ln x\right) \;\Rightarrow\; y' = \frac{1}{2}\sqrt{x}^{\,x}(1+\ln x)$

50. $y = (\ln x)^{\cos x} \;\Rightarrow\; \ln y = \cos x \ln(\ln x) \;\Rightarrow\; \dfrac{y'}{y} = \cos x\cdot\dfrac{1}{\ln x}\cdot\dfrac{1}{x} + (\ln\ln x)(-\sin x) \;\Rightarrow$

$y' = (\ln x)^{\cos x}\left(\dfrac{\cos x}{x\ln x} - \sin x \ln\ln x\right)$