Homework 18

Section 4.3:

2. (a) f is increasing on $(0,1)$ and $(3,7)$.

(b) f is decreasing on $(1,3)$.

(c) f is concave upward on $(2,4)$ and $(5,7)$.

(d) f is concave downward on $(0,2)$ and $(4,5)$.

(e) The points of inflection are $(2,2)$, $(4,3)$, and $(5,4)$.

6. (a) $f'(x) > 0$ and f is increasing on $(0,1)$ and $(3,5)$. $f'(x) < 0$ and f is decreasing on $(1,3)$ and $(5,6)$.

(b) Since $f'(x) = 0$ at $x = 1$ and $x = 5$ and f' changes from positive to negative at both values, f changes from increasing to decreasing and has local maxima at $x = 1$ and $x = 5$. Since $f'(x) = 0$ at $x = 3$ and f' changes from negative to positive there, f changes from decreasing to increasing and has a local minimum at $x = 3$.

14. (a) $f(x) = \cos^2 x - 2\sin x,\ \ 0 \le x \le 2\pi$. $\ f'(x) = -2\cos x\,\sin x - 2\cos x = -2\cos x\,(1 + \sin x)$. Note that $1 + \sin x \ge 0$ [since $\sin x \ge -1$], with equality $\Leftrightarrow \sin x = -1 \Leftrightarrow x = \frac{3\pi}{2}$ [since $0 \le x \le 2\pi$] $\Rightarrow$ $\cos x = 0$. Thus, $f'(x) > 0 \Leftrightarrow \cos x < 0 \Leftrightarrow \frac{\pi}{2} < x < \frac{3\pi}{2}$ and $f'(x) < 0 \Leftrightarrow \cos x > 0 \Leftrightarrow 0 < x < \frac{\pi}{2}$ or $\frac{3\pi}{2} < x < 2\pi$. Thus, f is increasing on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ and f is decreasing on $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$.

(b) f changes from decreasing to increasing at $x = \frac{\pi}{2}$ and from increasing to decreasing at $x = \frac{3\pi}{2}$. Thus, $f\left(\frac{\pi}{2}\right) = -2$ is a local minimum value and $f\left(\frac{3\pi}{2}\right) = 2$ is a local maximum value.

16. (a) $f(x) = x^2 \ln x \ \Rightarrow\ f'(x) = x^2(1/x) + (\ln x)(2x) = x + 2x\ln x = x(1 + 2\ln x)$. The domain of f is $(0,\infty)$, so the sign of f' is determined solely by the factor $1 + 2\ln x$. $f'(x) > 0 \Leftrightarrow \ln x > -\frac{1}{2} \Leftrightarrow x > e^{-1/2}$ [≈ 0.61] and $f'(x) < 0 \Leftrightarrow 0 < x < e^{-1/2}$. So f is increasing on $(e^{-1/2}, \infty)$ and f is decreasing on $(0, e^{-1/2})$.

(b) f changes from decreasing to increasing at $x = e^{-1/2}$. Thus, $f(e^{-1/2}) = (e^{-1/2})^2 \ln(e^{-1/2}) = e^{-1}(-1/2) = -1/(2e)$ [≈ -0.18] is a local minimum value.

18. (a) $f(x) = x^4 e^{-x} \ \Rightarrow\ f'(x) = x^4(-e^{-x}) + e^{-x}(4x^3) = x^3 e^{-x}(-x + 4)$. Thus, $f'(x) > 0$ if $0 < x < 4$ and $f'(x) < 0$ if $x < 0$ or $x > 4$. So f is increasing on $(0,4)$ and decreasing on $(-\infty, 0)$ and $(4,\infty)$.

(b) f changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 4$. Thus, $f(0) = 0$ is a local minimum value and $f(4) = 256/e^4$ is a local maximum value.

48. $f(x) = \dfrac{e^x}{1 - e^x}$ has domain $\{x \mid 1 - e^x \ne 0\} = \{x \mid e^x \ne 1\} = \{x \mid x \ne 0\}$.

(a) $\displaystyle\lim_{x\to\infty} \frac{e^x}{1 - e^x} = \lim_{x\to\infty} \frac{e^x/e^x}{(1 - e^x)/e^x} = \lim_{x\to\infty} \frac{1}{1/e^x - 1} = \frac{1}{0 - 1} = -1$, so $y = -1$ is a HA.

$\displaystyle\lim_{x\to-\infty} \frac{e^x}{1 - e^x} = \frac{0}{1 - 0} = 0$, so $y = 0$ is a HA. $\displaystyle\lim_{x\to0^+} \frac{e^x}{1 - e^x} = -\infty$ and $\displaystyle\lim_{x\to0^-} \frac{e^x}{1 - e^x} = \infty$, so $x = 0$ is a VA.

(b) $f'(x) = \dfrac{(1 - e^x)e^x - e^x(-e^x)}{(1 - e^x)^2} = \dfrac{e^x[(1 - e^x) + e^x]}{(1 - e^x)^2} = \dfrac{e^x}{(1 - e^x)^2}$. $f'(x) > 0$ for $x \ne 0$, so f is increasing on $(-\infty, 0)$ and $(0, \infty)$.

(c) There is no local maximum or minimum.