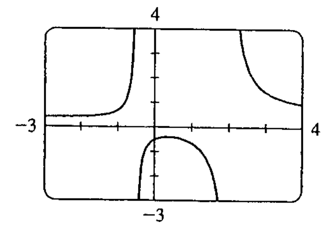
Homework 6

Section 2.6:

42. $\lim_{x\to\infty} \frac{x^2+1}{2x^2-3x-2} = \lim_{x\to\infty} \frac{\dfrac{x^2+1}{x^2}}{\dfrac{2x^2-3x-2}{x^2}} = \lim_{x\to\infty} \frac{1+\dfrac{1}{x^2}}{2-\dfrac{3}{x}-\dfrac{2}{x^2}} = \frac{\lim_{x\to\infty}\left(1+\dfrac{1}{x^2}\right)}{\lim_{x\to\infty}\left(2-\dfrac{3}{x}-\dfrac{2}{x^2}\right)}$

$= \frac{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \dfrac{1}{x^2}}{\lim_{x\to\infty} 2 - \lim_{x\to\infty} \dfrac{3}{x} - \lim_{x\to\infty} \dfrac{2}{x^2}} = \frac{1+0}{2-0-0} = \frac{1}{2}$, so $y = \frac{1}{2}$ is a horizontal asymptote.

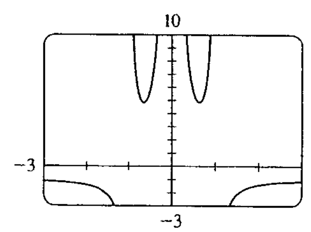
$y = f(x) = \frac{x^2+1}{2x^2-3x-2} = \frac{x^2+1}{(2x+1)(x-2)}$, so $\lim_{x\to(-1/2)^-} f(x) = \infty$ because as $x \to (-1/2)^-$ the numerator is positive while the denominator approaches 0 through positive values. Similarly, $\lim_{x\to(-1/2)^+} f(x) = -\infty$, $\lim_{x\to 2^-} f(x) = -\infty$, and $\lim_{x\to 2^+} f(x) = \infty$. Thus, $x = -\frac{1}{2}$ and $x = 2$ are vertical asymptotes. The graph confirms our work.



44. $\lim_{x\to\infty} \frac{1+x^4}{x^2-x^4} = \lim_{x\to\infty} \frac{\dfrac{1+x^4}{x^4}}{\dfrac{x^2-x^4}{x^4}} = \lim_{x\to\infty} \frac{\dfrac{1}{x^4}+1}{\dfrac{1}{x^2}-1} = \frac{\lim_{x\to\infty}\left(\dfrac{1}{x^4}+1\right)}{\lim_{x\to\infty}\left(\dfrac{1}{x^2}-1\right)} = \frac{\lim_{x\to\infty}\dfrac{1}{x^4} + \lim_{x\to\infty} 1}{\lim_{x\to\infty}\dfrac{1}{x^2} - \lim_{x\to\infty} 1}$

$= \frac{0+1}{0-1} = -1$, so $y = -1$ is a horizontal asymptote.

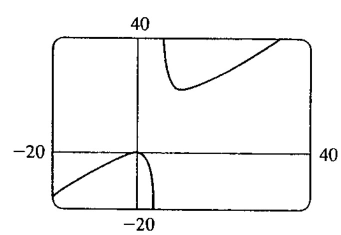
$y = f(x) = \frac{1+x^4}{x^2-x^4} = \frac{1+x^4}{x^2(1-x^2)} = \frac{1+x^4}{x^2(1+x)(1-x)}$. The denominator is zero when $x = 0, -1$, and 1, but the numerator is nonzero, so $x = 0$, $x = -1$, and $x = 1$ are vertical asymptotes. Notice that as $x \to 0$, the numerator and denominator are both positive, so $\lim_{x\to 0} f(x) = \infty$. The graph confirms our work.



45. $y = f(x) = \frac{x^3-x}{x^2-6x+5} = \frac{x(x^2-1)}{(x-1)(x-5)} = \frac{x(x+1)(x-1)}{(x-1)(x-5)} = \frac{x(x+1)}{x-5} = g(x)$ for $x \neq 1$.

The graph of g is the same as the graph of f with the exception of a hole in the graph of f at $x = 1$. By long division, $g(x) = \frac{x^2+x}{x-5} = x + 6 + \frac{30}{x-5}$.

As $x \to \pm\infty$, $g(x) \to \pm\infty$, so there is no horizontal asymptote. The denominator of g is zero when $x = 5$. $\lim_{x\to 5^-} g(x) = -\infty$ and $\lim_{x\to 5^+} g(x) = \infty$, so $x = 5$ is a vertical asymptote. The graph confirms our work.

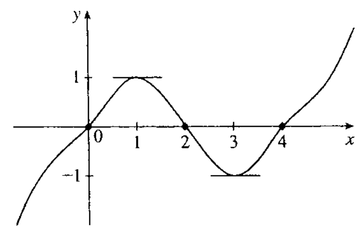


Section 2.7:

18. Since $g(5) = -3$, the point $(5, -3)$ is on the graph of g. Since $g'(5) = 4$, the slope of the tangent line at $x = 5$ is 4. Using the point-slope form of a line gives us $y - (-3) = 4(x - 5)$, or $y = 4x - 23$.

20. Since $(4, 3)$ is on $y = f(x)$, $f(4) = 3$. The slope of the tangent line between $(0, 2)$ and $(4, 3)$ is $\frac{1}{4}$, so $f'(4) = \frac{1}{4}$.

22. We begin by drawing a curve through the origin with a slope of 1 to satisfy $g(0) = 0$ and $g'(0) = 1$. We round off our figure at $x = 1$ to satisfy $g'(1) = 0$, and then pass through $(2, 0)$ with slope -1 to satisfy $g(2) = 0$ and $g'(2) = -1$. We round the figure at $x = 3$ to satisfy $g'(3) = 0$, and then pass through $(4, 0)$ with slope 1 to satisfy $g(4) = 0$ and $g'(4) = 1$. Finally we extend the curve on both ends to satisfy $\lim_{x\to\infty} g(x) = \infty$ and $\lim_{x\to-\infty} g(x) = -\infty$.



36. By Equation 5, $\lim_{x\to\pi/4} \dfrac{\tan x - 1}{x - \pi/4} = f'(\pi/4)$, where $f(x) = \tan x$ and $a = \pi/4$.

48. (a) $f'(5)$ is the rate of growth of the bacteria population when $t = 5$ hours. Its units are bacteria per hour.

(b) With unlimited space and nutrients, f' should increase as t increases; so $f'(5) < f'(10)$. If the supply of nutrients is limited, the growth rate slows down at some point in time, and the opposite may be true.