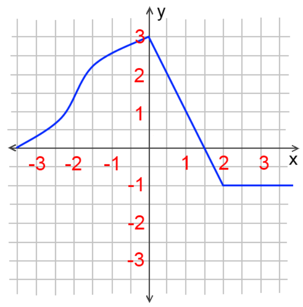
In-Class Work Solutions for April 25th

Part 1:

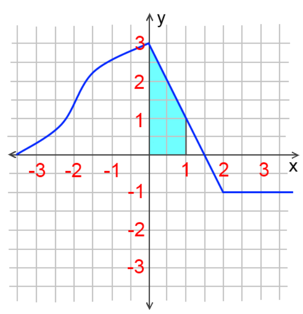
1. For the $f(x)$ in the picture below, calculate the following integrals:



(a) $\displaystyle\int_0^1 f(x)\,dx.$

Solution:

By definition, this requires us to calculate the shaded-in blue area in the following picture:



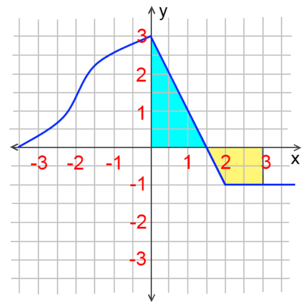
This can clearly be split up as the area of a triangle plus the area of a square. Thus,

$$\int_0^1 f(x)\,dx = \frac{1}{2}\cdot 2\cdot 1 + 1\cdot 1 = \boxed{2}$$

(b) $\displaystyle\int_0^3 f(x)\,dx.$

Solution:

By definition, this requires us to calculate the shaded-in blue area minus the shaded-in yellow area in the following picture:



Working these out,

$$\text{Blue area} = \frac{1}{2} \cdot 3 \cdot \frac{3}{2} = \frac{9}{4}$$
$$\text{Yellow area} = \frac{1}{2} \cdot 1 \cdot \frac{1}{2} + 1 = \frac{5}{4}$$

Thus,

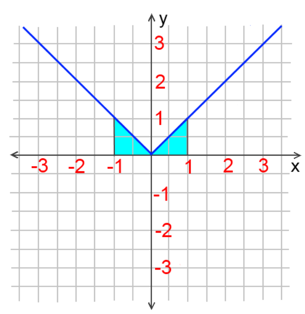
$$\int_0^3 f(x)\,dx = \frac{9}{4} - \frac{5}{4} = \frac{4}{4} = \boxed{1}$$

2. Use a sketch of the function $f(x) = |x|$ to calculate:

   (a) $\displaystyle\int_{-1}^{1} |x|\,dx$.

   Solution:

   This is the shaded-in blue area in the following picture:



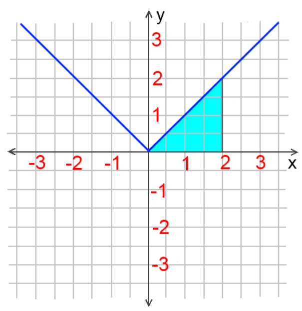
Thus,

$$\int_{-1}^{1} |x|\,dx = \frac{1}{2}\cdot 1\cdot 1 + \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2} + \frac{1}{2} = \boxed{1}$$

(b) $\int_0^2 |x|\,dx$.

Solution:

This is the shaded-in blue area in the following picture:



Thus,

$$\int_0^2 |x|\,dx = \frac{1}{2}\cdot 2\cdot 2 = \boxed{2}$$

Part 2:

1. For the following questions, you don't need to evaluate the sums – just write them down. Estimate the integral $\int_0^1 x^2\,dx$ using:

   (a) 5 rectangles and right endpoints. Make sure to draw a picture of what's going on. Is this an overestimate or an underestimate?

   Solution:

   I'm not going to draw a picture here, but you really should if you aren't sure where anything comes from. Here, the estimate is:

   $$\boxed{\int_0^1 x^2\,dx \approx \frac{1}{5}\cdot\frac{1}{25} + \frac{1}{5}\cdot\frac{4}{25} + \frac{1}{5}\cdot\frac{9}{25} + \frac{1}{5}\cdot\frac{16}{25} + \frac{1}{5}\cdot 1}$$

   (b) Can you guess what the estimate using 10 rectangles and right endpoints would be, using part (a) and the example from class, without drawing a picture?

   Solution:

The estimate is:

$$\boxed{\int_0^1 x^2\,dx \approx \frac{1}{10}\cdot\frac{1}{100}+\frac{1}{10}\cdot\frac{4}{100}+\frac{1}{10}\cdot\frac{9}{100}+\cdots+\frac{1}{10}\cdot\frac{81}{100}+\frac{1}{10}\cdot 1}$$

(Here, I put $\cdots$ in the middle in order not to have to write out all the ten terms – it should be clear what they are.)