In-Class Work Solutions for April 27th

Part 1: For the following questions, you don't need to evaluate the sums –
just write them down.

1. Estimate the integral $\int_\pi^{2\pi} \cos(x)\, dx$ using

   (a) Four rectangles and the left endpoint rule. Please draw the picture
   with the rectangles.

   Solution:

   The estimate is

   $$\int_\pi^{2\pi} \cos(x)\, dx \approx \frac{\pi}{4}\cos(\pi) + \frac{\pi}{4}\cos\left(\frac{5\pi}{4}\right) + \frac{\pi}{4}\cos\left(\frac{6\pi}{4}\right) + \frac{\pi}{4}\cos\left(\frac{7\pi}{4}\right)$$

   (b) Ten rectangles and the left endpoint rule. Write down the sum; you
   don't need to draw the picture if you can figure it out without.

   Solution:

   The estimate is

   $$\int_\pi^{2\pi} \cos(x)\, dx \approx \frac{\pi}{10}\cos(\pi) + \frac{\pi}{10}\cos\left(\frac{11\pi}{10}\right) + \cdots + \cdots + \frac{\pi}{10}\cos\left(\frac{19\pi}{10}\right)$$

   (c) One hundred rectangles and the left endpoint rule. Don't write down
   every term in the sum – write down enough terms so you can see the
   pattern.

   Solution:

   The estimate is

   $$\int_\pi^{2\pi} \cos(x)\, dx \approx \frac{\pi}{100}\cos(\pi) + \frac{\pi}{100}\cos\left(\frac{101\pi}{100}\right) + \cdots + \cdots + \frac{\pi}{100}\cos\left(\frac{199\pi}{100}\right)$$

2. Estimate $\int_1^4 e^x \, dx$ using three rectangles and the midpoint rule.

   Solution:

   The estimate is

   $$\int_1^4 e^x \, dx \approx 1 \cdot e^{1.5} + 1 \cdot e^{2.5} + 1 \cdot e^{3.5}$$
   $$= \boxed{e^{1.5} + e^{2.5} + e^{3.5}}$$