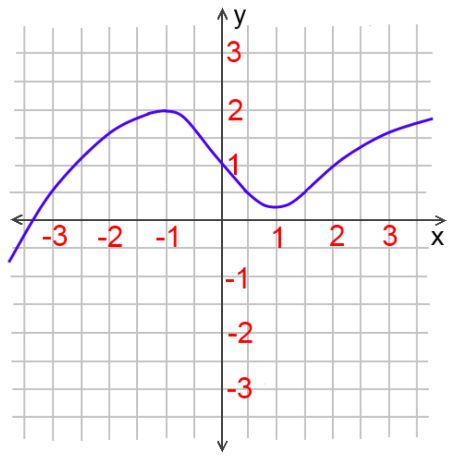
In-Class Questions for February 10th

Part 1:

1. For the $f(x)$ in the following graph:



   (a) Estimate $f'(0)$ and $f'(2)$.

   Solution:

   Drawing the tangent line to the curve at $x = 0$, we see that the slope is approximately -1. Therefore,

   $$\boxed{f'(0) \approx -1}$$

   Similarly, drawing the tangent line at $x = 2$, the slope looks to be somewhere between $\frac{1}{2}$ and 1. So a good estimate would be

   $$\boxed{f'(2) \approx \frac{2}{3}}$$

   Note that these really aren't exact! As long as you got similar answers, and you understand the logic, you're fine.

   (b) Graph $f'(x)$ on the empty set of axes.

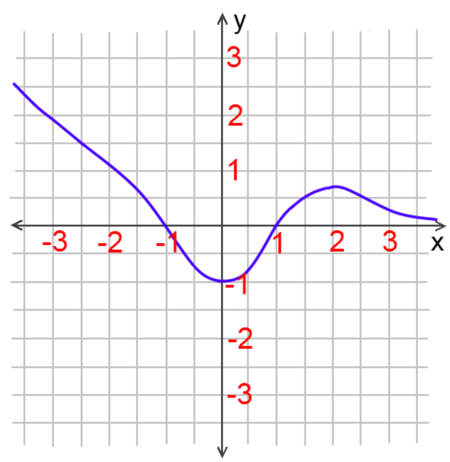
Solution:

Let's analyze what's going on before graphing $f'(x)$. First of all, the tangent is horizontal at $x = -1$ and $x = 1$. This means the the graph of $f'(x)$ contains the points $(-1, 0)$ and $(1, 0)$. Furthermore, the estimates from part (a) imply that the graph also contains the points $(0, -1)$ and $(2, 2/3)$.

Let us now also examine the behavior of $f'(x)$. For $x \leq -1$, the slope is positive and decreasing. This means $f'(x)$ is a decreasing function for $x \leq -1$. For x between -1 and 0, the slope is negative and decreasing, then for x between 0 and 1 the slope is negative and increasing.

Continuing, for x between 1 and 2 the slope looks positive and increasing. Then it looks like at 2, the slope is the biggest it gets (that is, the tangent line is the steepest) in that interval, and then the slope starts to decrease again.

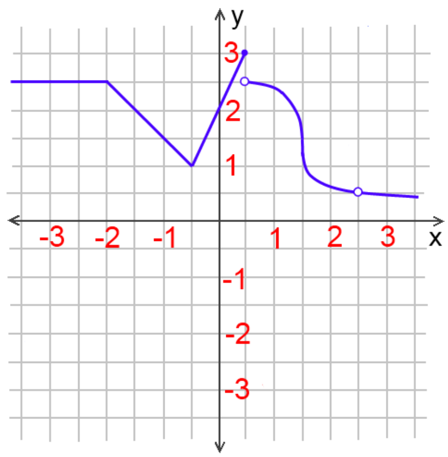
Graphing all this, we get:



Compare this to the picture of the original graph, to see exactly what's going on!

Part 2:

1. For the $f(x)$ in the following graph, do the following:



   (a) State the points where $f(x)$ isn't differentiable.

   Solution:

   Here are the points where $f(x)$ isn't differentiable, in increasing order:

   - $x = -2$ (corner)
   - $x = -1$ (corner)
   - $x = 0.5$ (discontinuity)
   - $x = 1.5$ (vertical tangent)
   - $x = 2.4$ (discontinuity)

   (b) Graph $f'(x)$ on the empty axes below.

   Solution:

   Again, let us start with the analysis. First, note that in places where the graph is a straight line, the tangent line to the graph is very easy: it's that very same straight line. This means that the straight line segments have simple derivatives. Using this, we see that for $x \leq -2$, the derivative (slope) is 0; for x between -2 and -0.5 the derivative is -1; and for x between -0.5 and 0.5 the derivative is 2.

   Since the tangent to the right-hand piece at 0.5 is horizontal, the graph of $f'(x)$ to the right of 0.5 starts off at the point $(0.5, 0)$ (although this point is an open circle.) Then for x between 0.5 and

1.5, the derivative decreases without bound – since at 1.5, there's a vertical asymptote, we see that the slope goes off to $-\infty$. Therefore, $f'(x)$ has a vertical asymptote at 1.5.

Similarly, we see that as we approach 1.5 from the right, the slope is going off to $-\infty$. Examining what's happening for $x > -1.5$, we see that we start off at $-\infty$, and then the derivative increases but stays negative.

Graphing all this, we get: