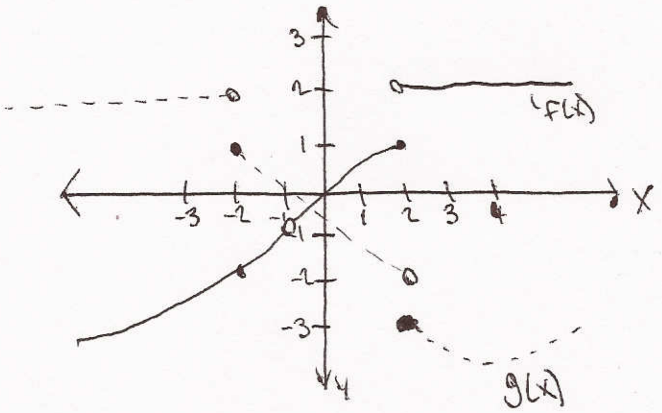
GROUP WORK
30 JAN 2012 (Monday)

① Let f and g be as in the following graph.



Calculate the following limits, or explain why they don't exist.

a) $\lim_{x \to -1} (f(x) + g(x))$

Limits from both sides of functions f(x) and g(x) approach the same point so they are separable:

$\lim_{x \to -1} (f(x) + g(x)) = \lim_{x \to -1} f(x) + \lim_{x \to -1} g(x)$

(evaluate) $= (-1) + (0) = \boxed{-1}$

b) $\lim_{x \to -2} (f(x) + g(x))$

Note: we first check to see if the limits are separable. f(x) does approach the same limit as x goes to negative two, but g(x) <u>does not</u> approach the same limit from the left as it does from the right. So, this limit is not separable.

– you're probably thinking, "well that's all dandy, but where do I go from here if I can't separate the limits of f(x) and g(x)?"

Remark: We can check to see if the limits of <u>both</u> f(x) and g(x) from the left equal the limits of f(x) and g(x) from the right. We do this by adding the two limits from both sides and seeing if they are equal.

$$\left(\lim_{x \to -2^-} f(x) + \lim_{x \to -2^-} g(x)\right) \overset{!}{=} \left(\lim_{x \to -2^+} f(x) + \lim_{x \to -2^+} g(x)\right)$$ for the limit to exist.

$$((-2) + (2)) \overset{!}{=} ((-2) + (1))$$

$0 \neq -1$, So the limit <u>does not exist</u>.

(if we got the result $-1 = -1$, then the limit would be -1.)

c) $\lim_{x \to 2} (f(x) + g(x)) =$ (can't separate, so check both sides simultaneously): Do limits from left add up and equal the sum of limits from the right?

$$\left(\lim_{x \to 2^-} f(x) + \lim_{x \to 2^-} g(x)\right) \overset{!}{=} \left(\lim_{x \to 2^+} f(x) + \lim_{x \to 2^+} g(x)\right)$$

neg. sign.

$$((1) + (-2)) \overset{!}{=} ((2) + (-3))$$

$\boxed{-1 = -1}$ So, the limit <u>does</u> exist and $\boxed{\lim_{x \to 2} (f(x) + g(x)) = -1.}$

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② Calculate:

$$\lim_{x\to1} \frac{x^2-1}{x^2+3x-4}$$

Factor numerator: $\frac{x^2-1}{x^2+3x-4} = \frac{(x+1)\cancel{(x-1)}}{(x+4)\cancel{(x-1)}} = \boxed{\frac{x+1}{x+4}}$ Now take its limit.

$$\lim_{x\to1} \frac{x+1}{x+4} = \frac{(1)+1}{(1)+4} = \boxed{\frac{2}{5}}$$

– Why do we need to get rid of common factors of the numerator and denominator before taking the limit?

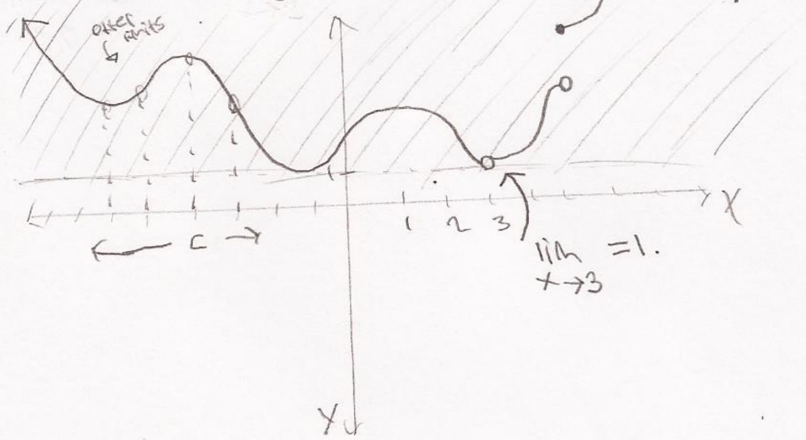
Well, this happens:

$$\lim_{x\to1} \frac{x^2-1}{x^2+3x-4} = \frac{(1)^2-1}{(1)^2+3(1)-4} = \frac{0}{0},$$ which is <u>not</u> the correct limit.

③ TRUE OR FALSE?

a) if $f(x) \geq 1$ on $(2,4)$, then $\lim_{x\to3} f(x) \geq 1$. <u>TRUE.</u>

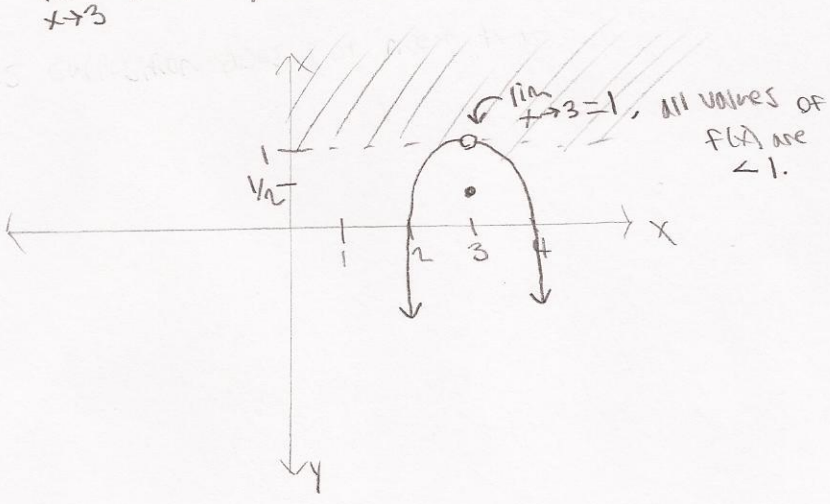
Why? Consider this graph.



Since all of our outputs, $f(x)$, are ≥ 1, the limit as $x \to 3$ will also fall somewhere between $[1, \infty]$.

The limit also doesn't have to approach 3; it could approach any value c that is in the domain $f(x)$.

b) if $\lim_{x\to3} f(x) \geq 1$, then $f(x) \geq 1$ on $(2,4)$. <u>False.</u>



The limit of $f(x)$ as $x \to 3$ is a particular case. It tells nothing of the behavior of $f(x)$ at points other than $x=3$. Actually, $f(3)$ doesn't even need to be ≥ 1 either.

Here, $f(3) = \frac{1}{2}$.

Limits describe behavior of functions <u>near</u> specified points.