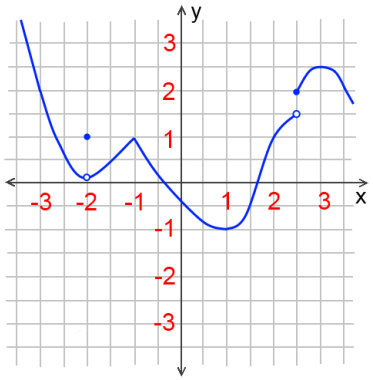
In-Class Questions for March 26th

1. Let $f(x)$ be given in the following picture:



   (a) Find the absolute minimum and absolute maximum values of $f(x)$ on $[-3, 3.5]$, as well as the values of x at which they are attained.

   (b) Find the values of x at which the local minimums are attained.

   (c) Find the values of x at which the local maximums are attained.

   (d) For each value of x in parts (b) and (c), state whether $f'(x)$ is 0 or doesn't exist.

2. Find all the critical values of $f(x) = (x+1)^{1/3}(x-2)^{2/3}$.

① (a) Find the absolute minimum and absolute maximum values of $f(x)$ on $[-3, 3.5]$, as well as the values of x at which they are attained.

Abs. Min: -1 at $x=1$; Abs. Max: 2.5 at $x=3$

(b) Find the values of x at which the local minimums are attained.

$x = 1$

(c) Find the values of x at which the local maximums are attained.

$x = -2$

$x = -1$

$x = 3$

(d) For each value of x in parts (b) and (c), state whether $f'(x)$ is 0 or DNE.

$x=1$; $f'(x)=0$

$x=-2$; $f'(x)=$ DNE

$x=-1$; $f'(x)=$ DNE

$x=3$; $f'(x)=0$

② Find all the critical values of $f(x) = (x+1)^{1/3}(x-2)^{2/3}$.

- Product Rule;

$$f(x) = (x+1)^{1/3}(x-2)^{2/3}$$

$$f'(x) = (x+1)^{1/3}\left[(x-2)^{2/3}\right]' + (x-2)^{2/3}\left[(x+1)^{1/3}\right]'$$

$$= (x+1)^{1/3}\left(\tfrac{2}{3}\right)(x-2)^{-1/3} + (x-2)^{2/3}\left(\tfrac{1}{3}\right)(x+1)^{-2/3}$$

- This appears daunting, but it's all algebra. Try to obtain a common denominator and simplify or your critical values will not correspond to the graph produced by the function.

$$= \frac{2(x+1)^{1/3}}{3(x-2)^{1/3}} + \frac{(x-2)^{2/3}}{3(x+1)^{2/3}} \quad \text{(common denominator)}$$

$$= \frac{2(x+1)^{1/3}}{3(x-2)^{1/3}} \cdot \frac{(x+1)^{2/3}}{(x+1)^{2/3}} + \frac{(x-2)^{2/3}}{3(x+1)^{2/3}} \cdot \frac{(x-2)^{1/3}}{(x-2)^{1/3}}$$

(expand numerator)

$$= \frac{2(x+1) + (x-2)}{3(x-2)^{1/3}(x+1)^{2/3}}$$

(collect terms)

$$= \frac{2x + 2 + x - 2}{3(x-2)^{1/3}(x+1)^{2/3}}$$

(simplify)

- Send negative exponents to the denominator.

$$= \frac{3x}{3(x-2)^{1/3}(x+1)^{2/3}}$$

(done.)

$$f'(x) = \frac{x}{(x-2)^{1/3}(x+1)^{2/3}}$$

→ Find critical points by letting $f'(x)=0$.

$$0 = \frac{x}{(x-2)^{1/3}(x+1)^{2/3}} \rightarrow$$

Values:

$x=0$, numerator is zero, so $f'(x)=0$.

$x=2$, denominator is zero, so $f'(x)=$ DNE.

$x=-1$, denominator is zero, so $f'(x)=$ DNE.