In-Class Work Solutions for March 9th

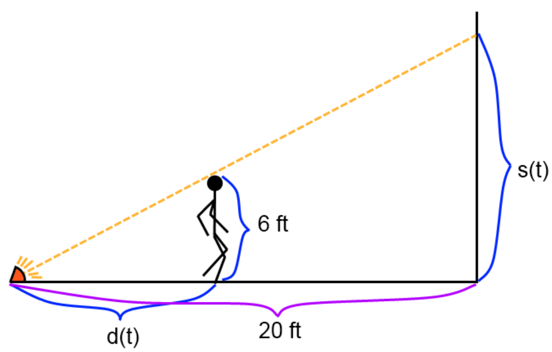
Part 1:

1. There is a light on the ground 20 feet away from a wall. A 6 foot man is walking away from the light towards the wall at 2 ft/sec. How fast is his shadow on the wall shrinking when the man is halfway between the light and the wall?
   Hint: Draw the picture and label everything carefully – that part is easiest to mess up! Also, make sure to follow the algorithm.

Solution:

Let us follow the algorithm.

1. **Draw a diagram:**



2. **Label the variables:** in the above picture, $d(t)$ is the distance of the man from the light, and $s(t)$ is the size of his shadow on the wall. (We label these as functions of t since they are all changing with time.)

3. **Write down information given using derivatives:** we know that the man is walking away from the lamp at 2 ft/sec. Since his distance from the lamp is increasing, we have that $d'(t)$ is positive; and since we're given the rate of change, we're given that

$$d'(t) = 2$$

4. **Write down what we want to find using derivatives:** we're looking for how quickly his shadow is shrinking when he's halfway between the light and the wall. At that point, he's precisely 10 feet away from the light; therefore, the question is

$$\text{What is } s'(t) \text{ when } d(t) = 10?$$

5. **Find a relationship**: it's clear from the picture that the triangle formed by the 6 foot man and $d(t)$ is similar to the triangle formed by $s(t)$ and the 20 foot distance of the wall to the light. Therefore, we get that

$$\frac{s(t)}{20} = \frac{6}{d(t)}$$

6. **Differentiate both sides of relationship with respect to t:** differentiating, (not forgetting the chain rule), we get

$$\frac{s'(t)}{20} = \frac{d(t) \cdot (6)' - (d(t))' \cdot 6}{d(t)^2} = -\frac{6d'(t)}{d(t)^2}$$

   Solving for $s'(t)$, we see that

$$s'(t) = 20 \cdot \left(-\frac{6d'(t)}{d(t)^2}\right) = -\frac{120d'(t)}{d(t)^2}$$

7. **Substitute information given:** we're asked about $s'(t)$ when $d(t) = 10$. We also know that $d'(t) = 2$. Plugging all this into the above equation, we get that

$$\begin{aligned} s'(t) &= -\frac{120d'(t)}{d(t)^2} = -\frac{120 \cdot 2}{10^2} = -\frac{240}{100} \\ &= -2.4\text{ft/sec} \end{aligned}$$

   Therefore, the answer is that

The man's shadow was shrinking at 2.4 feet per second.

   Note that the negative in the calculation above tells as that the shadow is shrinking – this is why there's no negative sign in the word answer.