GROUP WORK; May 2nd

PART 1:

① Find the most general antiderivatives of the following functions:

(a) $f(x) = x^2 + 2e^x$

$$\int f(x)dx = \int (x^2 + 2e^x)dx$$

$$= \int x^2 dx + 2\int e^x dx$$

Integrals distribute across sums, and pull out the constants to simplify the work.

$$= \boxed{\frac{x^3}{3} + 2e^x + C}$$

(b) $f(x) = 3\sin(2x) - \cos(x)$

Integrals distribute across sums; pull out the constants.

$$\int f(x)dx = \int (3\sin(2x) - \cos(x))dx$$

$$= 3\int \sin(2x)dx - \int \cos(x)dx$$

Use u-substitution to compute the integral of functions within functions.

$$= 3\int \sin(u)dx - \int \cos(x)dx, \quad u = 2x,$$

$$du = 2dx$$

$$\left(\tfrac{1}{2}\right)du = dx$$

replace dx by $\left(\frac{1}{2}\right)du$; that's where the $\left(\frac{1}{2}\right)$ comes from.

$$= 3\left(\tfrac{1}{2}\right)\int \sin(u)du - \sin(x)$$

$$= \left(\tfrac{3}{2}\right)(-\cos(u)) - \sin(x), \text{ but } u = 2x$$

$$= \boxed{\frac{-3\cos(2x)}{2} - \sin(x) + C}$$

② Find $f(t)$ such that $f'(x) = x^2 + 2e^x$, and $f(0) = 4$

$$f'(t) = t^2 + 2e^t$$

$$\int f'(t) = \int (t^2 + 2e^t)dt$$

$$= \int t^2 dt + 2\int e^t dt$$

$$f(t) = \frac{t^3}{3} + 2e^t + C,$$

Now use the initial condition $f(0) = 4$ to find C.

$$f(0) = 4 = \frac{0^3}{3} + 2e^0 + C$$

$$4 = 0 + 2 + C$$

$\boxed{2 = C}$ Plug 2 in for C.

$$\boxed{f(t) = \frac{t^3}{3} + 2e^t + 2}$$

③ Evaluate the following integrals:

(a) $\int_0^1 (x^2 + 2e^x)\,dx$

$= \left[\frac{x^3}{3} + 2e^x\right]_0^1$

$= \left(\frac{1^3}{3} + 2e^1\right) - \left(\frac{0^3}{3} + 2e^0\right)$

$= \frac{1}{3} + 2e - 2$

$= \boxed{2e - \frac{5}{3}}$

(b) $\int_0^{\pi/2} (3\sin(2x) - \cos(x))\,dx$

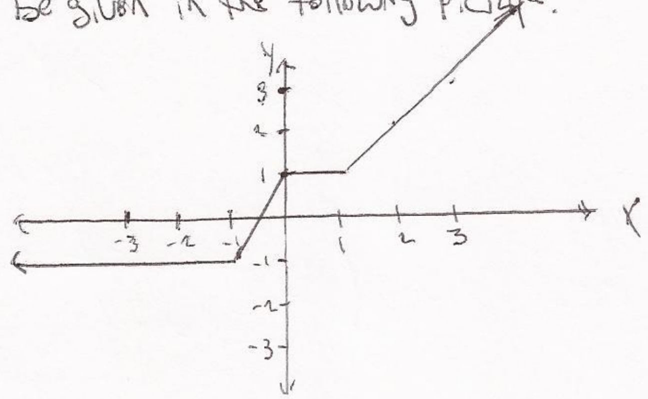
$= \left[-\frac{3\cos(2x)}{2} - \sin(x)\right]_0^{\pi/2}$

$= \left[\frac{-3\cos(2(\frac{\pi}{2}))}{2} - \sin(\pi/2)\right] - \left[\frac{-3\cos(0)}{2} - \sin(0)\right]$

$= \left[\frac{3}{2} - 1\right] - \left[\frac{-3}{2}\right]$

$= \boxed{2}$

PART 2: Let $f(t)$ be given in the following picture.



Define

$$g(x) = \int_1^x f(t)\,dt$$

① Calculate the following:

(a) $g(2) = \int_1^2 f(t)\,dt = \boxed{3/2}$

(b) $g(1) = \int_1^1 f(t)\,dt = \boxed{0}$

(c) $g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt$

$= -[0]$

$= \boxed{0}$

② Shade in the area corresponding to $g(2.1) - g(2)$. Now, Estimate the following quantity:

$$\frac{g(2.1) - g(2)}{(0.1)} = \boxed{2}; \quad \frac{2\cancel{(0.1)}}{\cancel{(0.1)}} = 2$$

Finally, guess the value of $\lim_{h \to 0} \frac{g(2+h) - g(2)}{h}$.

We plug in values close to zero to estimate this limit.

$$\lim_{h \to 0} \frac{g(2+h) - g(2)}{h} \approx \frac{g(2+0.1) - g(2)}{(0.1)} \approx \frac{g(2.1) - g(2)}{(0.1)} \approx \boxed{2}.$$