Midterm 1: Concepts to Review

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The first midterm will cover Sections 1.5, 1.6, 2.2, 2.3, 2.5, 2.6, 2.7, 2.8, and 3.1 – basically everything we did in lecture up until Monday, February 13th. In addition, any precalculus concepts that we've been using in lecture and on the homework will be tested implicitly!

1. Exponentials (Section 1.5)

   - The definition of $f(x) = a^x$.
   - Laws of exponents (and using them to solve problems)

   $$a^{x+y} = a^x a^y,\ a^{x-y} = \frac{a^x}{a^y},\ a^{xy} = (a^x)^y,\ (ab)^x = a^x b^x$$

   - Graphs of exponential functions

2. Inverse Functions and Logs (Section 1.6)

   - One-to-one functions
   - The definition of $f^{-1}(x)$ if $f(x)$ is one-to-one, domain and range of f^{-1}, graphing f^{-1} given the graph of f
   - Cancellation equations:

   $$f(f^{-1}(x)) = x,\ f^{-1}(f(x)) = x$$

   - Finding an explicit formula for the inverse of a function
   - Logarithms, logarithm laws, the number e and the natural log (ln):

   $$\log_a(xy) = \log_a(x) + \log_a(y),\quad \log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y),$$
   $$\log_a(x^r) = r\log_a(x),\quad \log_a(x) = \frac{\ln x}{\ln a}$$

   - Inverse trigonometric functions: definitions and graphs

3. The limit of a function (Section 2.2)

   - The concept of a limit: what it means when

   $$\lim_{x\to a} f(x) = L$$

- Using the graph of a function to determine a limit
- One-sided limits (knowing what the following mean):

$$\lim_{x\to a^+} f(x) = L, \lim_{x\to a^-} f(x) = L$$

- If $f(x) \le g(x)$ near (except possibly at) a, and both the limits exist, then

$$\lim_{x\to ax} f(x) \le \lim_{x\to a} g(x)$$

- The Squeeze Theorem: if $f(x) \le g(x) \le h(x)$ near (except possibly at) a, and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then

$$\lim_{x\to a} g(x) = L$$

- Infinite limits: the meanings of

$$\lim_{x\to a} f(x) = \infty, \lim_{x\to a} f(x) = -\infty$$

  and variants with one-sided limits, such as $\lim_{x\to a^+} f(x) = \infty$.
- Vertical asymptotes of functions

4. Calculating limits using limit laws (Section 2.3)

- If c is a constant, and $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then:

$$\begin{aligned}
\lim_{x\to a}(f(x)+g(x)) &= \lim_{x\to a} f(x) + \lim_{x\to a} g(x) \\
\lim_{x\to a}(f(x)-g(x)) &= \lim_{x\to a} f(x) - \lim_{x\to a} g(x) \\
\lim_{x\to a}(cf(x)) &= c\lim_{x\to a} f(x) \\
\lim_{x\to a}(f(x)g(x)) &= \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) \\
\lim_{x\to a} \frac{f(x)}{g(x)} &= \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \text{ if } \lim_{x\to a} g(x) \neq 0
\end{aligned}$$

- Other laws that follow from above in Section 2.3 (see textbook for full list)
- Being able to use the above laws to calculate limits
- Knowing when *not* to use these limit laws: if $\lim_{x\to a} f(x)$ or $\lim_{x\to a} g(x)$ don't exist, we can't use these. Remember that just because these don't exist does NOT mean that, say, $\lim_{x\to a}(f(x)+g(x))$ doesn't exist– you just can't use the laws to calculate it!

5. Continuity (Section 2.5)

- The definition of being continuous at a point: $f(x)$ is continuous at a if

$$\lim_{x\to a} f(x) = f(a)$$

- Similar definitions for $f(x)$ being continuous at a from the left and from the right.
- Using the graph of a function to determine where it's continuous
- If f and g are continuous at a, then the following functions are also continuous at a:

$$f+g,\ f-g,\ fg,\ \frac{f}{g} \text{ if } g(a) \neq 0$$

- Knowing functions that are continuous everywhere on their domain, and using this fact to calculate limits. These include: polynomials, rational functions, root functions, exponentials, logarithms, trig functions, inverse trig functions.

6. Limit at infinity (Section 2.6)

   - The meanings of

   $$\lim_{x\to\infty} f(x) = L,\ \lim_{x\to-\infty} f(x) = L$$

   - Horizontal asymptotes of functions
   - Rules such as

   $$\lim_{x\to\infty} \frac{1}{x^r} = 0$$

   for rational numbers $r > 0$.
   - Manipulating expressions in order to calculate limits at infinity

7. Calculating derivatives using limits (Sections 2.7)

   - $f'(a)$ is defined to be the slope of the tangent line to $y = f(x)$ at the point $(a, f(a))$.
   - $f'(a)$ is also the instantaneous rate of change of $f(x)$ at $x = a$.
   - The limit definition of the derivative is

   $$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

   - Finding the equation of a tangent line to $y = f(x)$ at $(a, f(a))$ using the derivative.

8. The derivative as a function (Section 2.8)

   - Just like above, the definition of $f'(x)$ is

   $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

   This is a function of x.

- What it means for a function to be differentiable at a and on an interval
- How a function can fail to be differentiable (some possibilities: a corner, a disconuity, or a vertical tangent)
- Higher derivatives: $f''(x)$ is the derivative of $f'(x)$, $f^{(n)}(x)$ is the nth derivative of $f(x)$ which is defined to be the derivative of $f^{(n-1)}(x)$.
- Graphing $f'(x)$ and $f''(x)$ given a graph of $f(x)$

9. Differentiation Rules (Section 3.1)

- Derivatives of constant functions and powers of x:

$$(c)' = 0$$
$$(x^n) = nx^{n-1}$$

- The sum, difference, and constant multiple rules:

$$(f(x) + g(x))' = f'(x) + g'(x)$$
$$(f(x) - g(x))' = f'(x) - g'(x)$$
$$(cf(x))' = cf'(x)$$

- The derivative of e^x:

$$(e^x)' = e^x$$