Midterm 2: Concepts to Review

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The second midterm covers Sections 3.2, 3.3, 3.4, 3.5, 3.6, 3.9, 3.10 and the beginning of 4.1 – everything we did in lecture starting on Monday, February 20th until Monday, March 26th. Material from the first exam will not appear *explicitly*, but since we've been building on that material, you should know it!

Note: Since we didn't have to compute derivatives using limits on the first exam, you will need to remember how to do it on this one.

1. The limit definition of the derivative:
   - The derivative $f'(x)$ is the slope of the tangent line to $y = f(x)$ at the point $(x, f(x))$.
   - Knowing the formula
   $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
   and using it to compute derivatives.
2. The Product and Quotient Rules (Section 3.2)
   - The product rule:
   $$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$
   - The quotient rule:
   $$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$
   Be careful with the order of the terms in the numerator!!
3. Derivatives of trig functions (Section 3.3:)
   - The main formulas:
   $$(\sin(x))' = \cos(x)$$
   $$(\cos(x))' = -\sin(x)$$

- You should know how to derive each of the below derivatives (although you can memorize and use them, unless explicitly asked to show it):

$$
\begin{aligned}
(\tan(x))' &= \sec^2(x) \\
(\cot(x))' &= -\csc^2(x) \\
(\csc(x))' &= -\csc(x)\cot(x) \\
(\sec(x))' &= \sec(x)\tan(x)
\end{aligned}
$$

4. The Chain Rule (Section 3.4)
   - If $F(x) = f(g(x))$, then

$$F'(x) = f'(g(x)) \cdot g'(x)$$

   - Alternatively with u, if $F(x) = f(u)$, then

$$F'(x) = f'(u) \cdot u'(x)$$

   - The following rule follows from the chain rule:

$$(a^x)' = \ln(a)a^x$$

5. Implicit Differentiation (Section 3.5)
   - Using the chain rule to find y' given a relationship between x and y: e.g., find $y' = \frac{dy}{dx}$ in terms of x and y if $x^2 + y^2 = xy$.
   - Substituting in the original relationship between x and y in order to simplify y'.
   - Derivatives of inverse trig functions, and knowing how to derive them using implicit differentiation:

$$
\begin{aligned}
(\arcsin(x))' &= \frac{1}{\sqrt{1-x^2}} \\
(\arccos(x))' &= -\frac{1}{\sqrt{1-x^2}} \\
(\arctan(x))' &= \frac{1}{1+x^2} \\
(\mathrm{arccot}(x))' &= -\frac{1}{1+x^2} \\
(\mathrm{arccsc}(x))' &= -\frac{1}{x\sqrt{x^2-1}} \\
(\mathrm{arcsec}(x))' &= \frac{1}{x\sqrt{x^2-1}}
\end{aligned}
$$

6. Derivatives of logarithmic functions and logarithmic differentiation (Section 3.6)

- The rule for differentiating $\ln(x)$:

$$(\ln(x))' = \frac{1}{x}$$

- Differentiating a log with another base:

$$(\log_a(x))' = \frac{1}{x\ln(a)}$$

- Figuring out the above derivatives using the algorithm for inverse functions we learned in class.
- Logarithmic differentiation: if $y = f(x)$ is written with a lot of products, quotients, and exponents, you can do the following:
  (a) Take the ln of both sides and simplify using log rules.
  (b) Differentiate implicitly with respect to x.
  (c) Solve for y', then substitute the original expression for y to get the answers in terms of x.
- An example where logarithmic differentiation would be useful: differentiate $y = \sin(x)^{\cos(x)}$.
- Make sure to use the log rules correctly! You can get all sorts of wrong answers by using 'identities' like $\ln(x + y) = \ln(x) + \ln(y), \ln(x)^r = r\ln(x)$, etc.

7. Related rates (Section 3.9)

- In related rates, all functions are in terms of time! When we write y' here, what we mean is $y'(t)$.
- Our algorithm for related rates from class:
  (a) Draw a diagram <u>corresponding to an arbitrary time.</u>
  (b) Label all the relevant variables. Make sure to label the variables which are changing with time as functions of t.
  (c) Write down the information given using derivatives.
  (d) Write down the information we want to find using derivatives.
  (e) Write down a relationship between the quantities in the question (possibly using the info given to decrease the number of variables.)
  (f) Differentiate both sides <u>with respect to t</u> using the chain rule, and solve for the required quantity.
  (g) Substitute information given (including the information about the instant we're interested in) into the equation from Step 6 – you may need to solve for some quantities at the instant given using the relationship found in Step 5.
- Common related rates problems:

(a) Two ships (cars, people, etc.) moving away from each other, their speed given – how quickly is the distance changing?
(b) Ladder sliding down a wall.
(c) Shadow problems (person walking away from streetlight, etc.)
(d) Volume and surface area growth problems.
(e) Point moving along a specified graph problems.

8. Linearizations (or linear approximations) (Section 3.10)

- The linearization $L(x)$ of $f(x)$ at $x = a$ is the function whose graph is the tangent line to $y = f(x)$ at $(a, f(a))$.
- For values of x that are close to a, $L(x)$ is close to $f(x)$; this allows us to use $L(x)$ to estimate $f(x)$. For example, we could estimate $\sqrt{4.1}$ using the linearization of $f(x) = \sqrt{x}$ at $x = 4$.
- Linearization can be used for word problems, such as 'Estimate the amount of paint needed to cover a 2 inch sphere with a 0.05 inch coat.'

9. Maximum and Minimum Values (Section 4.1):

- Definition of absolute minimum and maximum
- Definition of local minimum and maximum
- c is a critical point (or critical number) of f if it's in the domain of f, and $f'(c)$ is either 0 or doesn't exist.