MATH 408N PRACTICE FINAL

Name:_____

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TA session: _____

Show your work for all the problems. Good luck!

(1) Calculate the following limits, using whatever tools are appropriate. State which results you're using for each question.

(a) [5 pts] $\lim_{x\to 0} \dfrac{e^x + 1}{x^2 + 2}$

(b) [5 pts] $\lim_{x\to 0^+} x \ln(x)$

(c) [5 pts] $\lim_{x\to 2} \dfrac{\sqrt{2x} - 2}{x - 2}$

(2) Let the function $f(x)$ be defined piecewise as follows:

$$f(x) = \begin{cases} x^2 & x < 0 \\ 1 & x = 0 \\ x & x > 0 \end{cases}$$

(a) [5 pts] Does $\lim_{x \to 0} f(x)$ exist? If yes, calculate it; if not, explain why not.

(b) [5 pts] Is $f(x)$ continuous at 0? Explain why or why not.

(c) [5 pts] Does $f^{-1}(x)$ exist? If yes, sketch its graph; if not, explain why not.

(3) Find the following derivatives, using whatever tools you choose. You do NOT need to simplify your answer. Unless stated otherwise, answers should only be in terms of x.

(a) [5 pts] Find $f'(x)$, where $f(x) = \dfrac{\arctan(x)}{e^x + 1}$

(b) [5 pts] Find $f'(x)$, where $f(x) = \tan(x)^{2x^2}$.

(c) [5 pts] Find y' in terms of x and y if $2^x + y = \cos(xy) + x^3$.

(4) Solve the following questions:

(a) [5 pts] Use the limit definition of the derivative to calculate $f'(x)$, where $f(x) = \dfrac{1}{x^2}$.

(b) [5 pts] Find the equation of the tangent line to $f(x) = e^{x^2-1} + x$ at the point where $x = 1$.

(5) [10 pts] Two ships A and B start off at the same point, with ship A going north at the speed of 40 miles/hour, and ship B going east at the speed of 30 miles/hour. How quickly is the distance between them increasing after 2 hours?
Note: Please make sure to give descriptions for each of your variables, and to label them clearly in the picture.

(6) (a) [5 pts] If you wanted to find an estimate for $\sqrt{4.1}$, you could use the linearization of some function $f(x)$ at some $x = a$. What should you pick $f(x)$ and a to be?

(b) [5 pts] For the $f(x)$ and a you found in part (a), find the linearization $L(x)$ of $f(x)$ at a, and then use it to estimate $\sqrt{4.1}$.

(7) [10 pts] A rancher wishes to build a fence to enclose a rectangular pen having area 32 square yards. Along one side the fence is to be made of heavy duty material costing \$9 per yard, while the remaining three sides are to be made of cheaper material costing \$3 per yard. Determine the least cost of fencing for the pen.
Note: Please make sure to give descriptions for each of your variables, and to label them clearly in the picture. Also, don't forget to state the domain of your function, and explain which test you're using to optimize!

(8) Let $f(x) = \frac{x^2}{x^2-1}$.

(a) [5 pts] Find the intervals on which $f(x)$ is increasing and decreasing.

(b) [5 pts] Find the intervals on which $f(x)$ is concave up and concave down.

(c) [5 pts] Find the horizontal asymptotes of $f(x)$. For each asymptote, state whether it occurs at ∞ or $-\infty$.

(d) [5 pts] Find the vertical asymptotes of $f(x)$. For each vertical asymptote $x = a$, calculate $\lim_{x \to a^+} f(x)$ and $\lim_{x \to a^-} f(x)$.

(e) [5 pts] Use the information from the previous parts of the question to sketch the graph of $f(x)$.

(9) Let $f(x) = \dfrac{2x}{3} + x^{2/3}$.

(a) [5 pts] Find the critical numbers of $f(x)$.

(b) [5 pts] Find all the local minimumus and maximums of $f(x)$.

(c) [5x pts] What is the absolute maximum of $f(x)$ on $[-8, 0]$? State which method you're using.

(10) Solve the following questions:

(a) [5 pts] Find the most general $F(x)$ such that $F'(x) = e^{3x} + 2x + \dfrac{1}{4x}$.

(b) [5 pts] Find $\displaystyle\int_0^{\pi/2} (\cos(2x) - 3\sin(3x))\, dx$ using whatever tools you choose.

(c) [5 pts] Let $g(x) = \displaystyle\int_1^{\sin(x)} e^{t^2}\, dt$. Find $g'(x)$.

(11) Solve the following problems:

(a) [5 pts] Estimate the area under $y = x^2 + x$ from $x = 1$ to $x = 4$ using 3 rectangles and the left endpoint rule.

(b) [5 pts] Sketch a graph of $g(x) = |x| - 1$. Use it to calculate $\int_{-1}^{3} g(x)\,dx$.

(12) (a) [5 pts] The following sum approximates a certain integral using left endpoints. What is that integral, and how many rectangles are we using? When you find your answer, please check it by actually trying to do the approximation!

$$\frac{\pi}{4}\cos(\pi) + \frac{\pi}{4}\cos\left(\frac{5\pi}{4}\right) + \frac{\pi}{4}\cos\left(\frac{6\pi}{4}\right) + \frac{\pi}{4}\cos\left(\frac{7\pi}{4}\right)$$

(b) [5 pts] Write down the sum given in Part (a) using sigma notation.