MATH 408N PRACTICE MIDTERM 1

Name:_____

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TA session: _____

Show your work for all the problems. Good luck!

(1) (a) [5 pts] Solve for x if

$$2^{x+3} = 4^{3x-1}$$

(b) [10 pts] Let

$$f(x) = \frac{e^x}{e^x + 1}$$

Find a formula for $f^{-1}(x)$, and make your answer as simple as possible by using logarithm rules.

(2) [10 pts] Let $f(x)$ be defined as follows:

$$f(x) = \begin{cases} x & x \leq 0 \\ x^2 & 0 < x < 1 \\ 1 - x & 1 \leq x \end{cases}$$

Which values of a is this function continuous at? State your answer in interval notation. Make sure to show all the appropriate limit calculations and justify continuity for all stated values of a!

(3) Calculate the following limits. You must show all your work to get credit. State if you're using continuity.

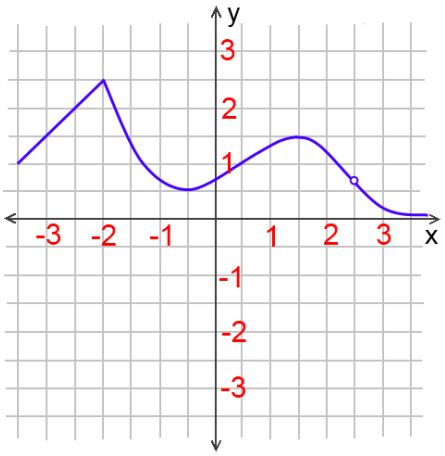
(a) [5 pts] $\lim_{x\to 0} \dfrac{\sqrt{3x+4}-2}{x}$

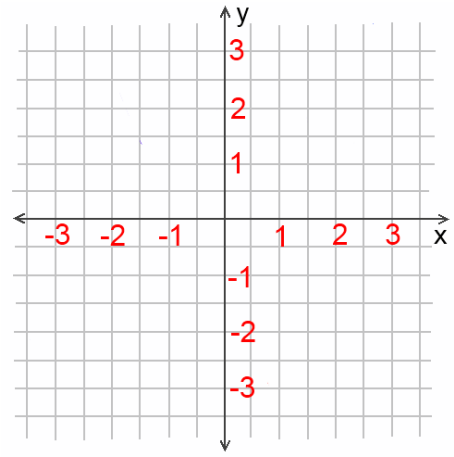
(b) [5 pts] $\lim_{x\to \infty} \dfrac{x^2+x+1}{2x^2-x+3}$

(c) [5 pts] $\lim_{x \to 1^-} \dfrac{x+1}{x^2 - 3x + 2}$

Hint: You might want to factor the denominator first...

(4) (a) [10 pts] Let $f(x)$ be given in the following graph. Sketch the graph of $f'(x)$ on the empty axes below. Make sure to estimate the values of $f'(x)$ carefully, and also to record whether $f'(x)$ is increasing or decreasing on the graph.





(b) [5 pts] Find $f'(x)$, if

$$f(x) = \frac{x^2 - 2x}{3x^3} + \frac{1}{2\sqrt{x}} + e^{x-1}$$

Use only the rules we have learned in class so far.