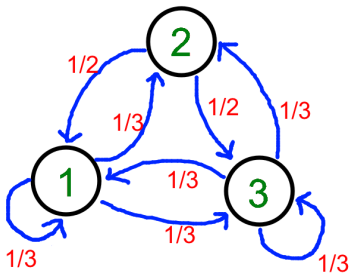
Quiz 2

1. Consider the following random walk:



(a) What is its transition matrix?

Solution: The transition matrix is

$$A = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}.$$

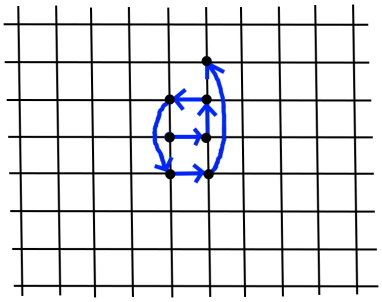
(b) What is $\mathbb{P}\{X_2 = 1 \,|\, X_1 = 1, X_0 = 3\}$?

Solution: Since we have a Markov chain, the distribution of X_2 depends *only* on the value of X_1, and this is given by the arrows in the above diagram (or the transition matrix if you prefer.) Thus,

$$\begin{aligned} \mathbb{P}\{X_2 = 1 \,|\, X_1 = 1, X_0 = 3\} &= \mathbb{P}\{X_2 = 1 \,|\, X_1 = 1\} \\ &= \mathbb{P}\{1 \to 1\} = \frac{1}{3} \end{aligned}$$

2. Consider the following random walk on the square grid. Start in the middle. At each step, flip a 4-sided die (I suppose it'd have to be a tetrahedron!) with N, S, E, W on its sides. Then, move the direction indicated on the die to the first location that has not yet been visited by the walk.

   Here's an example of a sequence of die flips and the corresponding path – $ENWSEN$:



   Is this walk a Markov chain? Why or why not?

   Solution: This walk is *not* a Markov chain. Since we move to the first location not yet visited by the walk, this depends on that path taken so far by the walk, and not only on the current location.