Theory of functions of a complex variable (M361)

T. Perutz

Fall 2012

- Theory of functions of a complex variable, UT Austin, Fall 2012
- Course number: M 361. Unique identifier: 56200
- Tuesday, Thursday, 2 - 3:30 pm, RLM 6.118
- Instructor: Timothy Perutz (Assistant Professor)
- Textbook: Jerrold E. Marsden and Michael J. Hoffman, *Basic Complex Analysis*, 3rd edition. Freeman, New York, 1999.
- Class webpage: www.ma.utexas.edu/users/perutz/CxVar2012.html

Aims. The main aims of the course are (i) to learn to work algebraically with complex number system; (ii) to learn the main results of complex analysis, both in their theoretic development and in examples; (iii) to learn to apply contour integration to problems concerning real numbers. The theoretical development, when told without shortcuts, is an unfolding narrative. I hope to tell much of this story, though I shall place less emphasis on proofs requiring skills developed in real analysis courses.

We shall cover material from chapters 1-4 of Marsden–Hoffman (see details below).

Assessment.

- **Homework** (15%).

  There will be homework assignments each week except when there is a test. The two lowest homework grades are dropped. Homework is due at the beginning of class on Thursdays, every week except when there is a test. Late work will not be accepted. You may discuss homework problems with others, but you should write out your solutions alone.

- **Final exam** (45%).

  Friday, December 14, 9:00 - 12:00 noon. There won't be an alternative date, so check this one now! Write in your calendar.

- **Two midterm exams** (20% each).

  In class, Thursday Oct 2, Thursday Nov 8. Put the dates in your calendar.

Further class policies.

- **Grading.** I will convert your weighted average to a (plus/minus) letter grade at the end of the semester. The conversion from numerical to letter grades will not be based on a pre-determined scale. However, to get an A, you will have to demonstrate fluency in most of the topics of the course.
- **Missed tests.** In case of a test missed because of illness or a serious family emergency, confirmed by a letter from an appropriate authority, I will shift the weight from the missed test to the final exam.
- **Disabilities.** Students with disabilities may request appropriate academic accommodations from the Division of Diversity and Community Engagement, Services for Students with Disabilities, 512-471-6259. Please contact them as early as possible in the semester.
- **Religious holidays.** If your observance of religious holidays clashes with the schedule for classes, tests and homework deadlines, I will make reasonable allowances provided that you notify me early. Please tell me by the 12th class day that this is an issue, and additionally notify me at least 14 days before each specific class, homework deadline or test that you need to miss.
- **Academic dishonesty** will not be tolerated, and will result in a failing grade for this course. If you are feeling overwhelmed by the demands on your time, or by the difficulty of the class, I will make time to talk it through with you. The wrong thing to do is to take a shortcut by cheating (for instance, plagiarizing work).
- **In class** I expect your attention. I expect you to write down what I write on the board. It's a good idea to make a note of things I say but don't write on the board, too. Texting, instant messaging, and catching up on sleep, are not acceptable in class; I may ask you to leave if you do these things.

Syllabus and schedule

Warning: the schedule is approximate! This is my first time teaching M361, so I can only estimate how long things will take. The section references are to the Marsden–Hoffmann text listed above.

Part 1: Complex numbers and functions of a complex variable

Date	Topic	Section
Aug 30	Complex numbers. Conjugation; $z\bar{z} = \|z\|^2$.	1.1–2
Sep 4	Polar form. De Moivre's theorem. Roots. Cauchy–Schwarz and triangle inequalities.	1.2
Sep 6	The exponential and trigonometric functions. Euler's formula. Complex logarithms; branches. Powers.	1.3
Sep 11	Open and closed sets; basic properties of continuous functions.	1.4
Sep 13	Holomorphic functions. The chain rule and inverse function theorem.	1.5
Sep 18	Conformality; the Cauchy–Riemann equations.	1.5
Sep 20	Harmonic functions and harmonic conjugates.	1.5
Sep 25	Examples of holomorphic functions; derivatives of standard functions.	1.6
Sep 27	Review.	
Oct 2	**First midterm test.**	

Part 2: Cauchy's theorem and its consequences

Oct 4	Contour integrals.	2.1
Oct 9	Statements of Cauchy's theorem and the deformation theorem. Subtleties in the statements. Relation to Green's theorem.	2.2
Oct 11	Simple connectivity and antiderivatives.	2.2
Oct 16	Cauchy for rectangles and in a disc.	2.3
Oct 18	Cauchy for nullhomotopic curves; deformation theorem.	2.3
Oct 23	Cauchy integral formula. Winding number.	2.4
Oct 25	Cauchy formula for higher derivatives. Morera's theorem	2.4
Oct 30	Liouville's theorem and the fundamental theorem of algebra.	2.4
Nov 1	Maximum modulus principle.	2.5
Nov 6	Review.	
Nov 8	**Second midterm test.**	

Part 3: Power series and contour integrals

Nov 13	Convergence of series.	3.1
Nov 15	Power series. Term-by-term differentiation. Taylor's theorem	3.2
Nov 20	Laurent series. Classification of singularities.	3.3
Nov 27	Residues. Removable singularities. The residue theorem.	4.1–2
Nov 29	Definite integrals via contour integration. Jordan's lemma.	4.3
Dec 4	More examples of contour integration, including Fourier transforms.	4.3–4
Dec 6	Review.	