A note on trigonometric identities

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There are, in my opinion, five trigonometric identities that are worth memorizing.

The first is the Pythagorean identity

$$\cos^2 x + \sin^2 x = 1 \tag{1}$$

that says that $(\cos x, \sin x)$ lies on the unit circle.

The second and third say that cosine is an even function and sine an odd function:

$$\cos(-x) = \cos x, \tag{2}$$

$$\sin(-x) = -\sin x. \tag{3}$$

If you know what the graphs of sine and cosine look like, you can make sure you're getting these two the right way round.

The fourth and fifth are the angle addition identities for sine and cosine:[1]

$$\sin(x + y) = \sin x \cos y + \cos x \sin y \tag{4}$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y. \tag{5}$$

If you're using one of these two and you're not sure you remembered the signs correctly, make sure that your identity works when $x = 0$ and also when $y = 0$.

If you know these five, you can work out all the other trig identities you need. With a little practice, you can do so quickly.

Try it: using only the above identities, establish

- a relationship between $\tan^2 x$ and $\sec^2 x$;
- a formula for $\sin 2x$ in terms of $\sin x$ and $\cos x$;
- a formula for $\cos 2x$ in terms of $\sin x$ and $\cos x$;
- a formula for $\cos 2x$ in terms of $\sin x$ alone;
- a formula for $\cos 2x$ in terms of $\cos x$ alone;
- a formula for $\tan(x + y)$ in terms of $\tan x$ and $\tan y$;
- a formula for $\cos(x + y) + \cos(x - y)$.

Another strategy is, of course, to memorize a long list of identities. That is at best a short-term strategy: a long list is unlikely to remain in your memory over a period of years. Those majoring in engineering and the natural sciences will likely find that they need trig formulas from time to time. Rather than relying on memory (or looking things up when needed), it is far better to hard-wire the five formulas above, and learn the skill of deriving the rest.

[1] Note for those who have studied complex numbers: it is easy to derive the angle-addition formulas for sine and cosine from the memorable fact that $e^{ix}e^{iy} = e^{i(x+y)}$ and Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$. The angle-addition formulas can also be proved using direct geometric arguments.