Research Methods in Mathematics
Homework 1

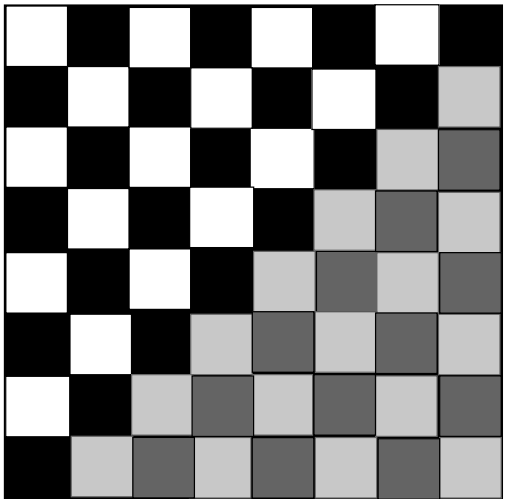
T. PERUTZ

Due at the beginning of class, Thursday September 2. Remember, you can discuss this homework with me and with our TA. *If you don't understand a question, please ask!*

I'll label each homework question with a category: "learning concepts", "using methods from class", "extending concepts from class", or "problem" (the last kind will occasionally be tough nuts).

I'll usually try to balance the different types of questions, but in this first assignment the questions don't involve any course material, and all of them are marked "problem".

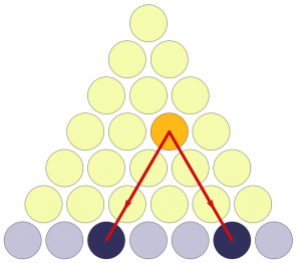
(1) *Problem.* An ordinary 8×8 chessboard has 28 squares below the diagonal (shaded gray in the picture).



(a) Explain why the number of squares below the diagonal on an $(n+1)\times(n+1)$ chessboard is $n(n+1)/2$.
(b) Use your answer to (a) to show that

$$1 + 2 + \cdots + n = n(n+1)/2.$$

(2) *Problem.*



(a) How many different ways are there to choose two items from $n+1$? Justify your answer. (*The order of the items is considered unimportant, so for instance, there are 3 ways to choose 2 items from 3.*)

(b) The picture represents another proof that $1+2+\cdots+n = n(n+1)/2$. Explain how this proof works. (You may find (a) useful.)

(3) *Problem.*

(a) Let m and n be positive integers with $n > m$. Find a formula for the sum

$$m + (m+1) + (m+2) + \cdots + n,$$

justifying your answer.

(b) What is the sum of all positive integers less than 1000 which are divisible by 3?

(4) *Problem.*

(a) If an equilateral triangle has sides of length 1, what is its area?

(b) If a regular hexagon has sides of length 1, what is its area?

(c) If we define π to be the area of a circle of radius 1, explain why $\pi > 3\sqrt{3}/2 \approx 2.60$.

(d) By computing the area of a regular dodecagon (12 sides) that fits snugly inside a circle of radius 1, show that $\pi > 3$.

(e) *Extra credit: Can you prove a better lower bound for π than 3?*