M346 (92153), Homework #7

Due: 10:00am, Wednesday, Jun. 20

Instructions: Questions are from the book "Applied Linear Algebra, 2nd ed." by Sadun. Please show all your work, not only your final answer, to receive credit. Keep answers organized in the same order the problems have been assigned.

Long-time behavior and stability (5.5)

p. 125, #2, 5, 8

[Hint: For problem #8, express the coefficients of the characteristic polynomial of a 2×2 matrix in terms of its trace and determinant.]

Markov chains and stochastic matrices (5.6)

p. 133-135, #3, 4, 9, 10, 12, 14

[Hint: For problems #10, 12, 14, use a calculator or online program such as *Wolfram Alpha* to easily find eigenvalues and eigenvectors of these 4×4 transition matrices, and to simulate evolution of the Markov chain.]

In addition:

A) Recall the following definitions. A transition matrix A is *irreducible* if for each fixed i, j, there is a $k = k(i, j) \geq 1$ such that $\left(A^k\right)_{ij} > 0$ (i.e., states i and j communicate in k steps). A is *aperiodic* if for each fixed i, there is a $K \geq 1$ such that for all $k \geq K$, $\left(A^k\right)_{ii} > 0$ (i.e., state i communicates with itself in an unpatterned manner). A transition matrix A is *regular* if A^K has all its entries strictly positive for some $K \geq 1$.

   i. Show that if A is regular, then it is irreducible.

   ii. Show that if A is regular, then it is aperiodic. [Hint: First show that if A is regular, then $A^{K+1} = A^K A$ has all entries strictly positive.]

   iii. Now prove the converse. That is, show that if A is irreducible and aperiodic, then it is regular.

B) For p. 134, #12, 14: Are either of these transition matrices regular? If not, are they irreducible or aperiodic, or neither? Classify all possible stationary states of each of these Markov chains.