M362K (56310), Homework #8

Due: 12:30pm, Thursday, Mar. 24

Instructions: Please show all your work, not only your final answer, in order to receive credit. Please keep answers organized in the same order the problems have been assigned.

Discrete distributions (3.4)

1. Pitman, p. 217, #1

2. Pitman, p. 217, #2

3. Pitman, p. 218, #3

4. Pitman, p. 218, #9

5. Pitman, p. 219, #14

6. Pitman, p. 220, #18 *[Note: Write $G=2X$, where X has* geometric(p^2+q^2) *distribution. Why is this true?]*

7. Pitman, p. 250, #2

8. Pitman, p. 254, #26

Poisson distribution (3.5)

9. Pitman, p. 234, #2

10. Pitman, p. 234, #4 *[Note: Assume that the number of misprints per page has a Poisson distribution and that the number of pages having more than 5 misprints is binomially distributed. Then use the Poisson approximation for the binomial distribution.]*

11. Pitman, p. 234, #8

12. Pitman, p. 234, #10

13. Pitman, p. 236, #17

Calculus problems

14. (*Integration by parts*)

a) Using integration by parts $\left(\int_a^b u(x)v'(x)dx = u(x)v(x)|_{x=a}^b - \int_a^b u'(x)v(x)dx\right)$, show that

$$\int_0^\infty te^{-t}\,dt = 1.$$

b) Compute

$$\int_0^\infty t^2e^{-t}dt$$

by integrating by parts once and then using part (a).

c) Define the *gamma function* with parameter $r>0$ by

$$\Gamma(r) = \int_0^\infty t^{r-1}e^{-t}dt.$$

Applying integration by parts once, show that $\Gamma(r) = (r-1)\Gamma(r-1)$. Conclude that for any positive integer r, $\Gamma(r) = (r-1)!$ That is, the gamma function generalizes the definition of the factorial from positive integers to all positive real numbers.

15. (*Multiple integration over a 2-D domain*)

Let D be the triangle with vertices $(-1,0)$, $(1,0)$, and $(0,1)$. Suppose

$$f(x,y) = \begin{cases} c & \text{if } (x,y) \text{ in } D \\ 0 & \text{otherwise} \end{cases}.$$

a) If $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dx\,dy = 1$, what is c?

b) Let $g(x) = \int_{-\infty}^{\infty} f(x,\, y)\,dy$. Give an explicit expression for $g(x)$. Be careful to note for which $x \in \mathbb{R}$ the function g is zero, and where it is nonzero.

c) Let $h(y) = \int_{-\infty}^{\infty} f(x,y)dx$. Similarly to part (b), evaluate $h(x)$.