M427L (55200), Homework #7

Due: 12:00pm, Wednesday, Oct. 12

Instructions: Questions are from the book "Vector Calculus, 5th ed." by Marsden and Tromba. Please show all your work, not only your final answer, to receive credit. Keep answers organized in the same order the problems have been assigned.

Vector fields (4.3)

p. 293-294, #3, 4, 7, 8, 9, 10, 14, 15, 18, 20

In addition:

- Show that for a particle satisfying Newton's law with force given by a gradient vector field, energy is conserved. That is, suppose the particle trajectory $\boldsymbol{r}(t)$ satisfies $\boldsymbol{F}(\boldsymbol{r}(t)) = m\boldsymbol{a}(t)$. Then if $\boldsymbol{F}(\boldsymbol{r}) = -\nabla U(\boldsymbol{r})$, show that the total energy (sum of kinetic plus potential energies)

$$E(t) = \frac{1}{2}m\|\boldsymbol{v}(t)\|^2 + U(\boldsymbol{r}(t))$$

  is invariant in time.

- In the scenario above, are particle trajectories flow paths for the force field $\boldsymbol{F}$? Explain why or why not.

Divergence and curl (4.4)

p. 310-313, #2, 3, 5, 6, 7, 13, 14, 18, 31