M365C (Rusin) HW5 – due Thursday, Oct 03 2019

1. Prove this result which I asserted in class: if $\{a_n\}$ and $\{b_n\}$ are convergent sequences of real numbers, and if $b_n \neq 0$ for every n, then the sequence with $c_n = a_n/b_n$ is also convergent if $\lim b_n \neq 0$.

2. We use the notation $C^0[0,1]$ for the set of continuous functions defined on the interval $[0,1]$. In a previous homework, we defined a metric on $C^0[0,1]$: $d(f,g) = \int_0^1 |f(x)-g(x)|\,dx$. Now let $a_n \in C^0[0,1]$ be the function $a_n(x) = x^n$. Show that $\{a_n\}$ is a Cauchy sequence. Does it converge?

3. Let X be any set and let d be the discrete metric on X. Show that X is a complete metric space. (Hint: show that only boring sequences are Cauchy sequences!)

4. My calculator shows that both $\sin(22)$ and $\sin(355)$ are very close to zero. Is there a subsequence of the sequence $\sin(n)$ which converges to zero?

5. Let A be a positive real number, and consider the sequence with $x_0 = 1$ and, for all $n \in \mathbf{N}$, $x_{n+1} = x_n(2 - Ax_n)$. Show that this sequence converges, and find its limit.