1. Find the general solution of \( x^4y'' + 5x^3y' + 4x^2y = 1 \).

2. Sketch the solution to the differential equation

\[
\frac{dy}{dx} = y^4 + 4 \quad y(3) = 0
\]

Identify any critical points and inflection points, and explain why there are or are not any horizontal or vertical asymptotes.

3. Solve the differential equation

\[
(4xy + 2y^2 + 2x) \frac{dy}{dx} = x^2 + 2xy + 3y^2 + 2y \quad y(1) = -2
\]

Hint: there is an integrating factor \( \mu \) for which \( \frac{\partial \mu}{\partial x} = \frac{\partial \mu}{\partial y} \).

4. Solve the system

\[
\frac{dx}{dt} = y(x + y)^5, \quad \frac{dy}{dt} = x(x + y)^5, \quad x(0) = 1, \quad y(0) = 0
\]

(Hint: Add and subtract.)

5. The biharmonic equation from continuum mechanics is the fourth-order linear partial differential equation \( u_{xxxx} + 2u_{xxyy} + u_{yyyy} = 0 \). For partial credit, find a nonzero solution \( u(x, y) \) to this equation. For full credit, find a non-polynomial solution. For extra credit, find an infinite-dimensional vector space of solutions.

Answers will soon appear at http://www.math.utexas.edu/users/rusin/Bennett/.