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Putnam Prep week 3 Problem 9:

Proof:

We start by observing that there exists a student who solved at least 4 problems correctly. For if not, then each student would have solved no more than 3 problems correctly, yielding a total number of correctly solved problems no greater than 3 x 200 = 600. But this contradicts our assumption that each problem was solved correctly by at least 120 students.

Let P_1, … , P_6 denote the problems, and S_1, …, S_{200} denote the students who participated in the contest. Without loss of generality, we may choose S_1 to have correctly solved at least 4 problems, and reorder the problems such that S_1 correctly solved P_1, P_2, P_3, and P_4.

Since each problem was solved by at least 120 students, we know that there exist 120 students who solved P_5, and 120 students who solved P_6. By the pigeonhole principle, there exists a student S* who solved both P_5 and P_6. If $S_1 = S^*$, then S_1 taken with any other player will satisfy the desired result. If not, then the pair $\{S_1, S^*\}$ is a pair of players such that any question was solved correctly by at least one of the two players.