2) $\sum_{n=2}^{\infty} ln((n^3-1)/(n^3+1)) = \sum_{n=2}^{\infty} ln((n-1)(n^2+n+1)/(n+1)(n^2-n+1))$

$= \sum_{n=2}^{\infty} ln((n-1)/(n+1)) + \sum_{n=2}^{\infty} ln((n^2+n+1)/(n^2-n+1))$

$= \sum_{n=2}^{\infty} ln(n-1) - ln(n+1) + \sum_{n=2}^{\infty} ln(n^2+n+1) - ln(n^2-n+1)$

These sums telescope since $ln((n+1)^2-n+1) = ln(n^2+n+1)$, and $ln((n+2)-1) = ln(n+1)$.

Thus $\sum_{n=2}^{\infty} ln((n^3-1)/(n^3+1)) = ln(1) + ln(2) - ln(2^2-2+1) = ln(2) - ln(3)$.

3) Clearly $\sum_{j=0}^{\infty} x_j^2 > 0$.

In addition, $\sum_{j=0}^{\infty} x_j^2 < \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} x_i x_j = (\sum_{j=0}^{\infty} x_j)^2 = A^2$

So that $0 < \sum_{j=0}^{\infty} x_j^2 < A^2$.

WLOG $A = 1$.

Thus it suffices to show that $\forall s, \quad 0 < s < 1, \exists x_j$ such that $\sum_{j=0}^{\infty} x_j = 1$ and $\sum_{j=0}^{\infty} x_j^2 = s$.

Let $x_j = ar^j$ with $r = (1-s)/(1+s)$, and with $a = 1-(1-s)/(1+s)$.

Since $0 < s < 1$, $0 < (1-s)/(1+s) < 1$, so that $\sum_{j=0}^{\infty} x_j = (1-(1-s)/(1+s))/(1-(1-s)/(1+s)) = 1$.

Also, $x_j^2 = a^2(r^2)^j$, so that $\sum_{j=0}^{\infty} x_j^2 = \frac{(1-(1-s)/(1+s))^2}{1-((1-s)/(1+s))^2}$

Then calculation shows that $\frac{(1-(1-s)/(1+s))^2}{1-((1-s)/(1+s))^2} = s$.

6) Lemma: If $p(x) \geq 0 \, \forall x$, then if $p(x)$ has a root at a, it has a double root at a,

i.e., If $p(x) = q(x)(x-a)$ for some polynomial $q(x)$, then $q(a) = 0$.

Take $x_n < a$, $y_n > a$ such that $\lim x_n = \lim y_n = a$.

$q(x_n) \leq 0$ since $x_n - a < 0$ and $p(x_n) \geq 0$. Similarly, $q(y_n) \geq 0$.

By the continuity of $q(x)$, $\ q(a) = \lim q(x_n) = \lim q(y_n)$.

But $q(x_n) \leq 0$ and $q(y_n) \geq 0$, so that $\lim q(x_n) \leq 0$ and $\lim q(y_n) \geq 0$.

Thus $q(a) = 0$.

Let $M_1 = \text{Inf}\ \{p(x)\}$.

M_1 exists because $p(x)$ is bounded from below by 0 and by the least-upper-bound property of R.

Moreover, $\exists x_l$ such that $p(x_l) = M_1$. If $p(x)$ is constant, than $M_1 = p(0)$ so that it is trivial.

Note that for $x \to \pm\infty$, $p(x) \to \infty$.

Also, for any infimum of a set S, $\exists a_n \in S$ such that $\lim a_n = \text{Inf S}$.

Thus $\exists z_n$ such that $p(z_n) \to M_1$.

If z_n is bounded, then by the Bolzano Weierstrass theorem , there exists a subsequence

z_k such that $\lim z_k \to z$ for some z. By the continuity of $p(x)$,

$\lim p(z_k) = p(z) = M_1$.

But if z_n is not bounded, i.e $\lim z_n = \pm\infty$, then

$\lim p(z_n) = \infty$ which is clearly not a lower bound.

Let $D(x) = p(x) - M_1$.
Then by above, $\exists x_1$ such that $D(x_1) = 0$. Clearly $D(x) \geq 0$, so that by the lemma,
$D(x)$ has a double root at x_1, i.e., $D(x) = (x - x_1)^2 p_1(x)$ for some polynomial $p_1(x) \geq 0$.
Thus $p(x) = M_1 + (x - x_1)^2 p_1(x)$.
However, $p_1(x) \geq 0$ so that, by the exact same process as above,
$p_1(x) = M_2 + (x - x_2)^2 p_2(x)$ for $M_2 \geq 0, x_2 \in R$, and $p_2(x) \geq 0$ some polynomial.
In general, $p_{n-1}(x) = M_n + (x - x_n)^2 p_n(x)$.
Since $p(x)$ has finite order, $\exists m$ such that $p_m(x) = 0$.
Then repeatedly applying this relation,
$p(x) = M_1 + (x - x_1)^2 (M_2 + (x - x_2)^2 (M_3 + (x - x_3)^2 (\ldots (M_m + (x - x_m)^2 * 0) \ldots)))$
$= M_1 + M_2 (x - x_1)^2 + M_3 (x - x_1)^2 (x - x_2)^2 + \ldots + M_m (x - x_1)^2 (x - x_2)^2 \ldots (x - x_{m-1})^2$
$= (\sqrt{M}_1)^2 + (\sqrt{M}_2 (x - x_1))^2 + (\sqrt{M}_3 (x - x_1)(x - x_2))^2 + \ldots + (\sqrt{M}_m (x - x_1)(x - x_2) \ldots (x - x_{m-1}))^2$,
So that $p(x)$ is a sum of squares.