For illustration I present an answer just to problem 1b:

1. In the game of NIM a pile of N chips is diminished by two players who alternately remove a number of chips, where the number is chosen from a set M. The person who takes the last chip wins. Determine the values of N for which the first player wins if:

$$(a)\ M = \{1, 2, 4, 8, 16, \ldots\} \qquad\qquad (b)\ \text{(Hard!)}\ M = \{1, 3, 8\}$$

It takes some time to discover what numbers are "good" or "bad", but once there are enough numbers in the two camps we can discern a pattern. Proving it just requires a bit more organization. But this is the final claim:

Theorem: A player who faces a pile of N chips can ensure a win if $N \equiv 1, 3, 5, 7, 8, 9,$ or 10 (mod 11) by making a suitable move, but cannot ensure a win if $N \equiv 0, 2, 4,$ or 6 (mod 11), no matter what move he makes.

Proof: This is certainly true for small values of N, e.g. if $N = 1$, the player simply selects 1 chip and wins; if $N = 5$ then the player can select 1 chip, leaving the opponent with a pile of 4, and either of the opponent's two possible moves (taking 1 or taking 3) can be countered by the first player taking the rest of the pile.

So we prove the statement for all integers by ("strong") induction. Assuming the characterization of piles is correct for all piles with fewer than N chips in it, we prove what has been claimed for an N-chip pile.

If $N \equiv 1$ then the player should take just 1 chip; the opponent will then be faced with a pile having $N - 1 \equiv 0$ chips, and so by induction, the opponent cannot force a win for himself, because any of his possible moves leaves a pile (of size $N - 2$ or $N - 4$ or $N - 9$) for which the original player has a strategy that can win.

Similarly we handle the other winning cases:

N(mod 11)	1	3	5	7	8	9	10
best move	1	1	1	1	8	3	8
opponent's task (mod 11)	0	2	4	6	0	6	2

In every case this player can leave his opponent with an undesirable pile size.

On the other hand, if say $N \equiv 0$ then the player has no good moves. Whether he withdraws 1, 3, or 8 chips, the size of the pile will be $N - 1 \equiv 10, N - 3 \equiv 8,$ or $N - 8 \equiv 3$, which by induction are all sizes of piles that will allow his opponent to force a win (assuming the opponent is clever, of course!) More generally, the other losing cases are shown to *be* losers, no matter what move is taken:

	pick 1	pick 3	pick 8
$N \equiv 0$	10	8	3
$N \equiv 2$	1	10	5
$N \equiv 4$	3	1	7
$N \equiv 6$	5	3	9

In each of these four cases, each of the three available moves leaves the opponent with a number of chips from which he can force a win.

Generally an analysis of one of these "combinatorial games" requires this kind of a proof: for any of the configurations that's considered good we must indicate (at least) *one* winning move that reduces the game to a configuration that's bad for the opponent; and for any of the configurations that we wish to show are bad, we must show that *none* of the possible moves will reduce the configuration that's bad for the opponent.

It is plausible that given any set M of values in the NIM game, the "good" values of N fall into certain congruence classes, but that is not obvious (to me). Specifically when $M = \{1, a, b\}$ it was suggested that the congruence classes would all be modulo $a + b$, but again no proof is obvious.