Question) Friendship is a symmetric and non-reflexive relation. That is, if Alice is Bob’s friend, then Bob is Alice’s friend, and nobody can count themselves as a friend. Is it possible to have an isolated community of 30 people such that no two people have the same number of friends?

Solution: In isolated group of n people, a person can have 0, 1, …. n-1 friends(Total n options). If every person has different number of friends then each one of the person has to choose from the given n options. Suppose the combination is as below:

Person 1: 0 friends

Person 2: 1 friend

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Person n: n – 1 friends(Everyone is his friend except himself)

Person n has everyone as his friend but person 1 has no friends, so contradiction occurred and thus there must be some people with same numbers of friends.