Calculus Problems

October 10, 2020

Problem 4

Let $f(n) = \sum_{k=2}^{\infty} \frac{1}{k^n \cdot k!}$. Compute $\sum_{n=2}^{\infty} f(n)$.

$$\begin{aligned}
\sum_{k=2}^{\infty}\sum_{n=2}^{\infty} \frac{1}{k^n k!} &= \sum_{k=2}^{\infty} \frac{1}{k!} \sum_{n=2}^{\infty} \frac{1}{k^n} \\
&= \sum_{k=2}^{\infty} \frac{1}{k!} \cdot (\frac{1}{k-1} - \frac{1}{k}) \\
&= \sum_{k=2}^{\infty} \frac{1}{(k-1)!k(k-1)} - \sum_{k=2}^{\infty} \frac{1}{k!k} \\
&= \sum_{k=2}^{\infty} \frac{1}{(k-1)!} \cdot (\frac{1}{k-1} - \frac{1}{k}) - \sum_{k=2}^{\infty} \frac{1}{k!k} \\
&= \sum_{k=2}^{\infty} \frac{1}{(k-1)(k-1)!} - \frac{1}{k!k} - \frac{1}{k} \\
&= 1 - \sum_{k=2}^{\infty} \frac{1}{k} = 1 - (e-2) = \boxed{3 - e}
\end{aligned}$$