Here are some problems, some of them taken from actual Putnam exams, all of them really "just" calculus.

1. Evaluate $\displaystyle\int_0^{12} \frac{x^2}{x^2+(12-x)^2}\,dx$

2. Assuming $a>0$ and $b>0$, evaluate

$$\lim_{n\to\infty} n^{b-a}\frac{1^a+2^a+\ldots+n^a}{1^b+2^b+\ldots+n^b}$$

3. Let $f(x)=\dfrac{1}{4+x^4}+A$ where A is constant, and let $F(x)$ be an antiderivative of f. Find a value of A for which F has exactly one critical point.

4. Let $f(n)=\displaystyle\sum_{k=2}^{\infty}\frac{1}{k^n\cdot k!}$. Compute $\sum_{n=2}^{\infty} f(n)$.

5. Show that this improper integral converges:

$$\int_0^{\infty} \sin(x)\sin(x^2)\,dx$$

6. Let $f(x)=x^3-x^2$. For a given value of c, the graph of $f(x)$, together with the graph of $y=x+c$, split the plane up into regions. Suppose that c is such that exactly two of these regions have finite area. Find the value of c that minimizes the sum of the areas of these two regions.

7. Compute

$$\int_0^{\pi/2}\frac{dx}{(\sqrt{\sin(x)}+\sqrt{\cos(x)})^4}.$$

8. Find the maximum of $\int_0^1 f(x)^3\,dx$ given the constraints that $-1\le f(x)\le 1$ for all x and $\int_0^1 f(x)\,dx=0$.