"Gee! I'm a tree!" (Putnam Nov 18 2021)

1. A rectangle $H O M F$ has sides $H O=11$ and $O M=5$. A triangle $A B C$ has $H$ as the intersection of its altitudes, $O$ as the center of its circumscribed circle, $M$ as the midpoints of $B C$, and $F$ as the foot of the altitude from $A$. What is the length of $B C$ ?
2. Let $d_{1}, d_{2}, \ldots, d_{12}$ be real numbers in the open interval $(1,12)$. Show that there exist distinct indices $i, j, k$ such that $d_{i}, d_{j}, d_{k}$ are the side lengths of an acute triangle.
3. What is the maximum number of rational points that can be on a circle in $\mathbf{R}^{2}$ whose center is not a rational point? (A rational point is a point both of whose coordinates are rational numbers.)
4. Show that for any set of five points on a sphere there is a set of four of them that lie on a closed hemisphere.
5. Can an arc of a parabola inside a circle of radius 1 have length greater than 4 ?
6. A unit cube is positioned in $R^{3}$ in some orientation and projected projected onto the coordinate plane $\left\{x_{1}=0\right\}$. What is the largest possible area of this projection?
