Introduction to Applied Harmonic Analysis Fall 2016

Course: M 393C / CSE 396

Time/location: Tuesday & Thursday 9:30 - 11:00 AM (RLM 11.176)

Instructor: Rachel Ward **Office:** RLM 10.144

Email: rward@math.utexas.edu

Office Hours: T/Th 11AM-noon, or by appointment

Target Audience: Graduate students in Math, ECE, ICES, CS, DSSC, and advanced

undergraduate students

Class webpage: www.ma.utexas.edu/~rward/courses/ACHA16.htm

Course Objective: This course should serve as an introduction to mathematical building blocks from time-frequency analysis (e.g. Fourier series, wavelets, sampling theorems) that can be used for signal and image processing, numerical analysis, and statistics. The course will emphasize the connection between the analog world and the discrete world, and focus on approximation and compression of functions and data. We will also discuss recent advances in sparse representations and compressive sensing.

Prerequisites: Linear algebra (M341 or equivalent), probability (M362K or equivalent), real analysis (M365C or equivalent), functional analysis or second-semester real analysis, or consent of instructor.

Topics:

- Fourier Transforms, Sampling Theorems, Fourier Series
- Discrete Fourier Transform, Discrete Cosine Transform

- The Uncertainty Principle and Bandlimited Signals
- Frame Theory
- Tools using Time/Frequency Domain Partitioning
 - Local Cosine/Sine Transform
 - o Fast Laplace/Poisson Solvers
 - o Haar Wavelets
 - o Multiresolution Analysis
 - o Discrete Wavelet Transform
 - o Image Approximation / Compression
- Principal Component Analysis
- Sparsity / Compressive Sensing

Textbooks:

-Required:

• G. Folland: Fourier Analysis and its Applications, 1992.

- Optional:

- I. Daubechies: Ten Lectures on Wavelets, 1992.
- S. Foucart and H. Rauhut: A Mathematical Introduction to Compressive Sensing, Springer 2013.

Grading Scheme: 50% Homework / participation.

50% Final project

Homework: Homework will be assigned every three weeks, and include proof-based and Matlab-based exercises. Additional problems will be presented during lectures, and students will present solutions to these problems during class.

Tentative Schedule:

• Thursday, Aug. 25: Introduction/ Overview

Tuesday, Aug. 30: Basics of Fourier Transforms I
 Thursday, Sept. 1: Basics of Fourier Transforms II

• Tuesday, Sept. 6: *Uncertainty Principles*

• Thursday, Sept. 8: Discussion

Tuesday, Sept. 13: Discretization via Sampling
 Thursday, Sept. 15: Fourier Series on Intervals

• Tuesday, Sept. 20: Functions of Bounded Variation

• Thursday, Sept. 22: Discrete Fourier Transform

• Tuesday, Sept. 27: Fast Fourier Transform

• Thursday, Sept. 29: Poisson Summation Formula

Tuesday, Oct. 4: Compressive sensing
Thursday, Oct. 6: Compressive sensing
Tuesday, Oct. 11: Compressive sensing
Thursday, Oct. 13: Compressive sensing
Tuesday, Oct. 18: Compressive sensing

• Thursday, Oct. 20: Frame Theory

• Tuesday, Oct. 25: Wavelets/ Short time Fourier Transform

• Thursday, Oct. 27: Wavelets

Tuesday, November 1: Fast Wavelet Transform
 Nov 3. – Nov. 22: FINAL PRESENTATIONS

• Nov. 29 – Dec. 1: Whatever is left

^{**} Lectures will often follow the notes of Naoki Saito at https://www.math.ucdavis.edu/~saito/courses/ACHA.w14/lectures.html